PhyloBayes

A Bayesian software for phylogenetic reconstruction using mixture models

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1 Introduction

PhyloBayes is a Bayesian Markov chain Monte Carlo (MCMC) sampler for phylogenetic inference. The program will perform phylogenetic reconstruction using either nucleotide, protein, or codon sequence alignments. Compared to other phylogenetic MCMC samplers, the main distinguishing feature of PhyloBayes is the use of non-parametric methods for modeling among-site variation in nucleotide or amino-acid propensities.

PhyloBayes3.4, implements several probabilistic models:

- non-parametric models of site-specific rates, equilibrium frequency profiles or substitution matrices, using Dirichlet processes.
- Tuffley and Steel’s covarion model (Tuffley and Steel, 1998), as well as the mixture of branch length (mbl) models (Kolaczkowski and Thornton, 2008; Zhou et al., 2007).
- empirical mixtures of profiles, or of matrices.
- auto-correlated and non auto-correlated relaxed clocks for molecular dating

1.1 Modeling site-specific effects using non-parametric methods

Among-site variation in the rate of substitution is often modeled using a gamma distribution. Doing so, however, only models variation in the rate of evolution, and does not account for the fact that the different nucleotides or amino-acids might have uneven propensities across positions.

As a way of modeling state propensity variation along sequences, in PhyloBayes, the rate and also a profile—controlling nucleotide or amino acid equilibrium frequencies associated with the substitution process—are modeled as site-specific random variables. As has been shown in several previous works, accounting for such generalized among-site variation results in a better statistical fit and greater phylogenetic accuracy. Of particular interest to phylogenetic reconstruction, such models show a greater robustness to systematic errors such as long-branch attraction problems.

There are two ways site-specific propensities can be modeled. First, one can use a parametric model. In such models, the parametric form of the law describing the distribution of the site-specific feature under investigation is assumed known, up to a few parameters that will be estimated from the data. The best example is the use of a gamma distribution for modeling the distribution of relative rates of substitution across sites (Yang, 1994).
Alternatively, one can use non-parametric models: the overall shape of the distribution across sites is not specified a priori but is directly inferred from the data. It is thus more general than the parametric method. There are different non-parametric methods. A practical approach, often adopted in Bayesian inference, is the use of Dirichlet process mixtures (Ferguson, 1973). Formally, these are infinite mixtures, i.e. mixtures with a countably infinite number of components.

PhyloBayes uses Dirichlet processes for modeling sites-specific profiles (Lartillot and Philippe, 2004). Each site is thus given a frequency vector profile over the 20 amino-acids or the 4 bases. These are combined with a globally defined set of exchange rates, so as to yield site-specific substitution processes. The global exchange rates can be fixed to uniform values (the CAT-Poisson, or more simply CAT, settings), to empirical estimates (e.g. JTT, WAG or LG) or inferred from the data (CAT-GTR settings).

1.2 Empirical mixture models

The non-parametric models introduced above are flexible. They automatically estimate the distribution of site-specific effects underlying each dataset. But on the other hand, they may require a relatively large amount of data for reliable estimation.

An alternative that would be suitable for smaller alignments is offered by the so-called empirical mixture models. Unlike non-parametric mixtures, empirical models have a fixed, pre-determined set of components, which have been estimated on a large database of multiple sequence alignments. Classical empirical matrices, such as JTT, WAG and LG (Le and Gascuel, 2008) are a specific case of empirical model, with only one component. A few years ago, empirical mixtures of profiles were proposed (Quang et al., 2008; Wang et al., 2008), which are implemented in the current version of PhyloBayes. The user can also specify its own custom set of exchange rates or mixture of profiles.

1.3 Phylogenetic reconstruction

Samples approximately from the posterior distribution are obtained by Markov chain Monte Carlo (MCMC) and are then used to estimate marginal distributions, or expectations, over the parameters of interest. Concerning the topology of the phylogenetic tree, PhyloBayes works like usual Bayesian phylogenetic reconstruction programs and outputs a majority-rule posterior consensus tree. Conversely, one may be interested in the site-specific biochemical specificities that have been captured by the infinite mixture model, in which case mean
posterior site-specific profiles will be estimated from the MCMC output.

1.4 Molecular dating

PhyloBayes can be used for Bayesian molecular dating using relaxed molecular clocks. Several alternative clock relaxation models are available, including the log-normal (Brownian) auto-correlated model of Thorne et al. (1998), the CIR process of Lepage et al. (2007) and uncorrelated clocks (Drummond et al., 2006). Fossil calibrations can be provided as hard constraints (Kishino et al., 2001) or as soft constraints (Yang and Rannala, 2006).

1.5 Choosing a model

It is still difficult to get a general idea of the relative merits of the models, as it depends on the dataset and on the criterion used to measure the fit. But a few trends are observed, which can be used for establishing general guidelines.

1.5.1 Amino acid replacement models

Except for small datasets (less than 400 aligned positions), CAT-GTR is virtually always the model with highest fit among all models implemented in PhyloBayes. The CAT-Poisson model, which was the model initially developed in Lartillot and Philippe (2004), is less fit than CAT-GTR but generally more fit than single-matrix models on large and datasets, particularly when mutational saturation (multiple substitutions) is prevalent.

Concerning the posterior consensus tree, both CAT-GTR and CAT-Poisson are significantly more robust against long-branch attraction (LBA) artifacts, compared to all other models. However, in some cases, there are differences between the topologies obtained under CAT or CAT-GTR. The flat exchange rates of CAT (which were introduced mostly for the sake of computational efficiency) are not biologically realistic, suggesting that CAT-GTR would generally be a better option.

Empirical mixture models (C60 or WLSR5) may be appropriate for single gene datasets.

1.5.2 Nucleotide substitution models

The CAT-GTR model is a very good generic model also for DNA or RNA data, again probably better than CAT (which does not correctly handle differences in transition and transversion rates) but also, in most cases, better than classical nucleotide substitution models.
1.5.3 Rate across sites

PhyloBayes implements several models of rate variation across sites: the discrete gamma distribution (default model), the continuous gamma distribution and a Dirichlet process over site-specific rates (Huelsenbeck and Suchard, 2007).

The Dirichlet process appears to have a better fit than the more classical discrete gamma distribution with 4 categories. On the other hand, it may sometimes result in MCMC convergence and mixing difficulties (also true for the continuous gamma model). Thus far, we have not seen many cases where the Dirichlet process over the rates was resulting in a significantly different phylogeny, compared to the discrete gamma model.

1.5.4 Dataset size

In practice, the Dirichlet process mixture works best for alignments in the range of 1 000 to 20 000 aligned positions. Beyond 20 000 positions, convergence and mixing of the Monte Carlo progressively become challenging. As for the number of taxa, the MCMC sampler seems able to deal alignments with up to 100 taxa reasonably well.

A possible approach for analyzing very large multi-gene datasets (large in terms of the number of aligned positions) is to perform jackknife resampling procedures (although resampling procedures are not so easily justified from a Bayesian philosophical standpoint.). An example of gene-jackknife using PhyloBayes is described in Delsuc et al. (2008).
2 Input data format

2.1 Sequences

The main format recognized by PhyloBayes is a generalization of the PHYLIP format:

<number_of_taxa> <number_of_sites>
taxon1 sequence1...
taxon2 sequence2...
...

Taxon names may contain more than 10 characters. Sequences can be interrupted by space and tab, but not by return characters. They can be interleaved, in which case the taxon names may or may not be repeated in each block.

PhyloBayes automatically recognizes DNA, RNA, or protein alphabets. The following characters will all be considered equivalent to missing data: "-", “?”, “$”, “.”, “#”, “X”, “x”, as well as the degenerate bases of nucleic acid sequences (“B”, “D”, “H”, “K”, “M”, “N”, “R”, “S”, “V”, “W”, “Y”), and the “B” and “Z” characters for protein sequences. Upper or lower case sequences are both recognized, but the case matters for taxon names.

PhyloBayes can also be applied to datasets with arbitrary alphabets. The file should then be formatted as follows:

#SPECIALALPHABET
<number_of_taxa> <number_of_sites> <ALPHABET>
taxon1 sequence1...
taxon2 sequence2...
...

where the alphabet is specified by concatenating all characters, in one single word, e.g.:

#SPECIALALPHABET
2 4 ABCDE
taxon1 ACABEDE
taxon2 ACEBBDE

2.2 Trees

In some cases, an initial tree can be provided, or the program can be constrained to sample the posterior distribution of parameters under a specified tree topology, which will then remained fixed throughout the analysis. Trees should be provided in NEWICK format.
Taxon names should correspond to the names specified in the data matrix (case sensitive). If some names are present in the tree but not in the matrix, the corresponding taxa will be pruned out of the tree. That is, the spanning subtree containing all the taxa mentioned in the data matrix will be considered as the input tree. Conversely, if some taxa are present in the data matrix but not in the input tree, the program will exit with an error message.

2.3 Outgroups

For molecular dating, a tree needs to be specified, and its rooting matters. If a rooted tree is given as an input, it is used as such. If an unrooted tree is given, an outgroup has to be specified to the program. For this, the outgroup has to be written into a file, using the following format:

```
<number of taxa>
<taxon1>
<taxon2>
...
```

All the taxa making the outgroup should be specified. They should correspond to a split of the tree.

2.4 Calibrations

Molecular dating requires fossil calibrations to be specified. The format is as follows:

```
<ncalib>
<taxon1a> <taxon1b> <upper_limit> <lower_limit>
<taxon2a> <taxon2b> <upper_limit> <lower_limit>
...
```

The calibration given on each line will be enforced on the last common ancestor of the two specified taxa. Upper limits are older, and thus, should always specify larger numbers than lower limits. Upper or lower limits can be set equal to -1, in which case no limit is enforced. For example:

```
taxon1 taxon2 -1 50
```

means that the node of the last common ancestor of taxon1 and taxon2 should be older than 50 millions of years (Myr), but otherwise, does not have an upper constraint. Likewise:

```
taxon1 taxon2 70 -1
```
only specifies an upper constraint of 70 Myr, and no lower constraint. And finally:

taxon1 taxon2 70 50

specifies an upper and a lower constraint, thus the interval $[50, 70]$ Myr.

For a node that has both an upper and a lower constraint, it is important to specify both on the same line, thus as:

taxon1 taxon2 70 50

and not

taxon1 taxon2 -1 50
taxon1 taxon2 70 -1
3 General presentation of the programs

PhyloBayes is provided both as executable files (for linux x86-64) and as a C++ source code. Depending on the operating system running on your cluster, you may need to recompile the code. To this end, a simple Makefile is provided in the sources directory, and compiling with the make command should then work in most simple situations.

The CIR clock relaxation model (Lepage et al., 2007) requires functions from the gsl library. To make compiling as simple as possible, the CIR model is deactivated by default. In order to use this model, one should first uncomment a compilation instruction in the Makefile (at the very beginning of the file, remove the # in front of USE_GSL=-DGSL). This of course requires gsl to be installed on your machine.

The general philosophy of PhyloBayes is rather 'unixy': instead of having one single program that does everything, through standard input and output streams (i.e. keyboard and screen), several small programs have been written, working more or less like unix commands using input and output files. The detailed model and Monte Carlo settings can be controlled through options, starting with -.

Specifically, the following programs can be found in the package (for details about any of these programs, type the name without arguments):

- **pb**: the MCMC sampler.
- **readpb**: post-analysis program, for computing a variety of posterior averages.
- **bpcomp**: evaluates the discrepancy of bipartition frequencies between two or more independent runs and computes a consensus by pooling the trees of all the runs being compared.
- **tracecomp**: evaluates the discrepancy between two or more independent runs based on the summary variables provided in the trace files.
- **ppred**: posterior predictive analyses.
- **readdiv**: post-analysis program for relaxed clock analyses.
- **ancestral**: estimation of ancestral sequences and substitution histories.
- **cvrep/readcv/sumcv**: model comparison by cross-validation.

In this section, a rapid tour of the programs is proposed. A more detailed explanation of all available options for each program is given in the next section.
3.1 Running a chain (pb)

A run of the pb program will produce a series of points drawn from the posterior distribution over the parameters of the model. Each point defines a detailed model configuration (tree topology, branch lengths, nucleotide or amino-acid profiles of the mixture, etc.). The series of points defines a chain. To run the program:

```
pb -d <dataset> <chainname>
```

The `-d` option is for specifying the dataset. Many other options are available for specifying the model (see below). The default options are the CAT-Poisson model with discrete gamma (4 categories). Before starting, the chain will output a summary of the settings.

A series of files will be produced with a variety of extensions. The most important are:

- `<name>.treelist`: list of sampled trees;
- `<name>.trace`: the trace file, containing a few relevant summary statistics (log-likelihood, total length of the tree, number of components in the mixture, etc).
- `<name>.chain`: this file contains the detailed parameter configurations visited during the run and is used by `readpb` for computing posterior averages.

The chains will run as long as allowed. They can be interrupted at any time and then restarted, in which case they will resume from the last check-point (last point saved before the interruption). To soft-stop a chain, use `stoppb`:

```
stoppb <chainname>
```

Alternatively, you can open the `<name>.run` file and replace the 1 by a 0. Under linux, this can be done with the simple following command:

```
echo 0 > <chainname>.run
```

The chain will finish the current cycle before exiting. To restart an already existing chain:

```
pb <chainname>
```

Be careful not to restart an already running chain.
3.2 Checking convergence and mixing (bpcomp and tracecomp)

It is difficult to know beforehand how long a chain should run. Different datasets, or different models, may not require the same number of cycles before reaching convergence and may display very different mixing behaviors. In general, for larger datasets, each cycle will take more time, but also, more cycles will be needed before reaching convergence. Note also that the absolute number of cycles is not really a relevant measure of the quality of the resulting sample: update mechanisms for the mixture, the topology or the hyperparameters are not really comparable, and their mixing efficiency depends very much on the model, the data and the implementation. In the case of PhyloBayes, the MCMC sampler saves one point after each cycle. A cycle itself is made of a set of complex and integrated series of updates of the topology, the branch length or the substitution model (including the mixture), which are not easily compared with the number of generations realized by other phylogenetic samplers. Generally, a run under PhyloBayes provides good results for a total number of points of the order of 10 000 to 30 000, although again, this really depends on the datasets.

The best is therefore to rely on more objective measures, such as effective sample size and reproducibility of the results across independent runs started from different initial conditions.

In the case of the relatively complex infinite mixture models CAT and CAT-GTR, convergence and mixing should be carefully assessed both for the phylogenetic and for the mixture aspects of the model. Thus, one should make sure that posterior consensus trees are reproducible across independent runs, but also, that the trace plots of the summary statistics recorded in the trace file capturing various sub-components of the model (tree length, alpha parameter, number of occupied components of the infinite mixture, entropy of the mixture, entropy of exchangeabilities) appear to be at stationarity and to be reproducible across runs.

Convergence can first be visually assessed by plotting the summary statistics recorded in the trace file as a function of number of iterations. This can be done using simple linux utilities, such as gnuplot. Alternatively, the trace file of PhyloBayes is compatible with the Tracer program of the Beast software. Thus, you can use Tracer to check convergence and estimate effective sample size (tracecomp, introduced below, does similar things, albeit with a more primitive interface).

It is also good practice to run at least two chains in parallel and compare the samples obtained under these several independent runs. The can be done using the tracecomp program (for checking convergence of the continuous parameters of the model) and the bpcomp program (for assessing convergence in tree space). Both use a similar syntax:
bpcomp -x 1000 10 <chain1> <chain2>

Here, using a burn-in of 1000, and sub-sampling every 10 trees, the bpcomp program will output the largest (maxdiff) and mean (meandiff) discrepancy observed across all bipartitions. It will also produce a file (bpcomp.con.tre) with the consensus obtained by pooling all the trees of the chains given as arguments.

Note that bpcomp can be run on a single chain (in which case it will simply produce the consensus of all trees after burn-in). However, using bpcomp on multiple chains usually results in more stable MCMC estimates of the posterior consensus tree.

Some guidelines:

• maxdiff < 0.1: good run.
• maxdiff < 0.3: acceptable: gives a good qualitative picture of the posterior consensus.
• 0.3 < maxdiff < 1: the sample is not yet sufficiently large, and the chains have not converged, but this is on the right track.
• if maxdiff = 1 even after 10,000 points, this indicates that at least one of the runs is stuck in a local maximum.

Similarly,

tracecomp -x 1000 <chain1> <chain2>

will produce an output summarizing the discrepancies and the effective sizes estimated for each column of the trace file. The discrepancy $d$ is defined as $d = 2|\mu_1 - \mu_2|/(\sigma_1 + \sigma_2)$, where $\mu_i$ is the mean and $\sigma_i$ the standard deviation associated with a particular column and $i$ runs over the chains. The effective size is evaluated using the method of Geyer (1992). The guidelines are:

• maxdiff < 0.1 and minimum effective size > 300: good run;
• maxdiff < 0.3 and minimum effective size > 50: acceptable run.

3.3 Obtaining posterior consensus trees and parameter estimates

The readpb program is meant for estimating some key parameters of the model. As a simple example:

readpb -x 1000 10 <chainname>
will take one tree every ten, discarding the first 1000 trees. It will produce several additional files:

- `<chainname>_sample.general`, containing a few general statistical averages (mean posterior log likelihood, tree length, alpha parameter, number of modes, etc.)
- `<chainname>_sample.con.tre`: the majority-rule posterior consensus tree
- `<chainname>_sample.bplist`: the list of weighted bipartitions.

Various options allow one to compute several other posterior averages, such as site-specific rates and equilibrium frequency profiles (see below, detailed options).

### 4 Divergence times

PhyloBayes allows for molecular dating under several alternative relaxed clock models and priors on divergence times. In the following, we give a brief presentation of this feature of the program, while emphasizing the most important points to be careful about.

#### 4.1 A brief example

Using PhyloBayes, molecular dating only works under a fixed topology, which has to be specified using the -T option. The molecular dating option of `pb` is activated by specifying a model of clock relaxation or by specifying calibrations. Here is an example, taken from Douzery et al. (2004). Files are in the `data/moc` directory:

```
pb -d moc.ali -T moc.tree -r bikont.outgroup -cal calib -ln -rp 2000 2000 mocln1
```

Here, the program is given a protein sequence alignment (`-d moc.ali`), a tree file (`-T moc.tree`), an outgroup (`-r bikont.outgroup`, imposing a basal split between unikonts and bikonts), and a file containing 6 fossil calibrations (`-cal calib`) specified in millions of years. A log-normal autocorrelated relaxed clock (`-ln`) is assumed, together with a uniform prior on divergence times (default prior). A gamma prior of mean 2000 and standard deviation 2000 Myr (thus, an exponential of mean 2000 Myr) is specified for the age of the root.

PhyloBayes provides several relaxed clock models, either autocorrelated, such as the log normal clock (Thorne et al., 1998) or or the CIR process (Lepage et al., 2007), or uncorrelated, such as the white noise process (Lepage et al., 2007) or the uncorrelated
gamma multipliers (Drummond et al., 2006). See Lepage et al. (2007) for a more detailed review of all of these models. The autocorrelated lognormal model, the white noise and the uncorrelated gamma models are probably the most important models to use and compare with each other.

Calibrations should be given in a separate file (see 'Format' section). They are specified using the following option: `-cal <calibration_file>`. The time scale (the unit of time) is implicitly defined by the calibrations. Thus, if calibrations are in millions of years, 1 time unit = 1 Myr. If no calibration is specified, by convention, the total duration from root to tips is 1000 time units. By default, PhyloBayes implements hard calibrations, but soft constraints can also be set up, using the `-sb` option (which works only under the birth death prior). These soft constraints are implemented according to the method introduced by Yang and Rannala (2006), with some further adaptations in Inoue et al. (2010).

The prior on divergence times can be either uniform, birth-death, or Dirichlet. In addition, a prior on the age of the root should be specified. This can be done either by including a calibration interval for the root in the calibration file (which will amount to a uniform prior over the specified interval), or by using the `-rp` option, as done above in the example, so as to specify a gamma prior for the root age.

The prior proposed by the user on the age of the root and on the other nodes is the prior without calibrations. It may sometimes be very different from the prior conditional on the calibrations. Therefore, it is advised to run the program under the prior but with the calibrations, and check that the distributions obtained for the nodes of interest (and, in particular, for the root) indeed correspond to the intended prior – in particular, that they are sufficiently wide if they were meant to be non informative – before doing any posterior sampling. Running under the prior can be done using the `-prior` option. In our example:

```
 pb -d moc.ali -T moc.tree -r bikont.outgroup -cal calib -ln
    -rp 2000 2000 -prior -ncat 1 -f81 -dgam 1 moclnprior1
```

Note that, since the prior over divergence times is independent from the prior over the other parameters of the model, we can speed up computation by using the simplest possible substitution model (`-ncat 1 -f81 -dgam 1`, which is a F81 model without rate variation across sites). Once the MCMC has run, the induced prior distribution distribution over the age of the root, and more generally over the ages of the nodes of interest, can be visualized using `readpb`.

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5 Detailed options

5.1 pb

5.1.1 General options

-d <datafile>

specifies the multiple sequence alignment to be analyzed.

-dc

constant sites are removed. Note that the likelihood will not be properly conditioned on this removal, as it normally should be (as explained by Lewis, 2001). Using this option is therefore problematic.

-t <treefile>

forces the chain to start from the specified tree.

-T <treefile>

forces the chain to run under a fixed topology (as specified in the given file). In other words, the chain only samples from the posterior distribution over all other parameters (branch lengths, alpha parameter, etc.), conditional on the specified topology. This should be a bifurcating tree (see Input Data Format section).

-r <outgroup>

roots the tree using the specified outgroup. Current substitution models implemented in PhyloBayes are reversible, so this rooting option is useful only for clock models.

-s

by default, pb only saves the trees explored by the chain during MCMC. This is enough if the aim is to infer the tree topology. However, for a more detailed analysis, such as inferring site-specific rates or equilibrium frequency profiles, conducting cross-validation or posterior predictive checks, the complete parameter configurations visited by the MCMC should be saved. This can be done using the -s option. Fixed topology analyses (using the -T option) will automatically activate this option.
-f

forces the program to overwrite an already existing chain with same name.

-x <every> [<until>]

specifies the saving frequency and (optional) the number of points after which the chain should stop. If <until> is not specified, the chain will run as long as desired. By definition, -x 1 corresponds to the default saving frequency. In some cases, samples may be strongly correlated, in which case, if disk space or access is limiting, it would make sense to save points less frequently, say 10 times less often: to do this, one can use the -x 10 option.

5.1.2 Prior on branch lengths

-lexp

prior on branch lengths is a product of i.i.d. exponential distributions.

-lgam

prior on branch lengths is a product of i.i.d. gamma distributions (more flexible than the exponential, as it allows independent control of the mean and the variance of the distribution on branch lengths).

-meanlength <meanlength>

fixes the mean of the exponential (or gamma) priors to the specified value. Without this command, this mean is considered as a free parameter, endowed with an exponential prior of mean 0.1. In the case of the gamma prior, the variance/mean ratio is always considered as a free parameter, with an exponential prior of mean 1.

5.1.3 Rates across sites

-dgam <n>

specifies n categories for the discrete gamma distribution. Setting n = 1 amounts to a model without across-site variation in substitution rate.

-uni
uniform-rate across sites (same model as -dgam 1).

-cgam

the distribution of rates across sites is a continuous gamma distribution.

-ratecat

the distribution of rates across sites is inferred non-parametrically, using a Dirichlet process prior (as in Huelsenbeck and Suchard, 2007).

5.1.4 Relative exchangeabilities (exchange rates)

-poisson / -poi

exchange rates are all equal to 1. The model is then a mixture of Poisson (F81) processes.

-lg, -wag, -jtt, -mtrev, -mtzoa, -mtart

specifies empirical exchangeabilities.

-gtr

specifies a general time reversible matrix: exchangeabilities are free parameters, with prior distribution a product of independent exponential distributions of mean 1.

-rr <filename>

exchangeabilities are fixed to the values given in the specified file. The file should be formatted as follows:

[<ALPHABET>]
<rr1_2> <rr1_3> ... <rr1_20>
<rr2_3> <rr2_4> ... <rr2_20>
...<rr18_19> <rr18_20>
<rr19_20>

The order in which amino acids should be considered is specified on the first line ([<ALPHABET>]), with letters separated by spaces or tabs. This header should then be followed by the exchangeabilities in the specified order (separated by spaces or tabs or returns).
5.1.5 Profile mixture

~dp (or ~cat)

the distribution of equilibrium frequency profiles across sites is inferred non-parametrically, using a Dirichlet process prior (Lartillot and Philippe, 2004).

~ncat <n>

specifies a mixture of equilibrium frequency profiles of \( n \) components; the number of components is fixed whereas the weights and profiles are treated as random variables. Fixing the number of components of the mixture most often results in a poor mixing of the MCMC. The Dirichlet process usually has a much better mixing behavior.

~catfix <predef>

specifies a mixture of a set of pre-defined profiles (the weights are re-estimated). The argument <predef> can be either one of the following keywords: C20, C30, C40, C50, C60, which correspond to empirical profile mixture models (Quang et al., 2008); or WLSR5, which correspond to the model of Wang et al. (2008). Note that this latter model actually defines 4 empirical profiles, which are then combined with a fifth component made of the empirical frequencies of the dataset.

~catfix <filename>

specifies a mixture of a set of user-pre-defined profiles, where <filename> is the name of a file containing a set of profiles specified as follows:

[[<ALPHABET>]]
ncat
weight freq1 freq2 ... freq20
weight freq1 freq2 ... freq20
...

where <ncat> is the number of profiles, and each line following this number should be a set of 21 real numbers, defining a weight, and then a profile of equilibrium frequencies (separated by spaces or tabs). The order in which amino acids should be considered is specified on the first line ([[<ALPHABET>]]), with letters separated by spaces or tabs. Note that the weights are there only for historical reasons – they are re-estimated anyway.
5.1.6 Combining profiles and exchange rates

Any set of exchange rates can be combined with any of the three settings for the mixture. But the same set of exchange rates will be used by all components of the mixture. For instance, `-cat -gtr` makes an infinite mixture model whose components differ by their equilibrium frequencies but otherwise share the same set of relative exchange rates (themselves considered as free parameters). As another example, `-catfix w1r5 -jtt` defines the Wang et al. (2008) model: a model with 5 components, each of which is a matrix made from the relative exchange rates of the JTT matrix, combined with one of the 4 vectors of equilibrium frequencies defined by Wang et al. (2008), plus one vector of empirical frequencies.

The default model is `-cat -poi`, that is, a mixture of Poisson (F81) processes. Note that, if non-Poisson exchange rates are used, the mixture model is automatically deactivated by default. Thus, for instance, the `-gtr` option is equivalent to `-gtr -ncat 1`. In order to use a Dirichlet process combined with a GTR matrix of exchange rates, one has to explicitly use the `-gtr -cat` combination of options.

5.1.7 Matrix mixture models

`-qmm`

activates the QMM model, which is a mixture of matrices (thus, allowing for distinct sets of exchange rates and equilibrium frequencies for each component of the mixture). MCMC mixing is very challenging under this model configuration.

`-qmmfix ul2 / -qmmfix ul3`

defines the empirical mixture with 2 or 3 components (Le et al., 2008).

`-qmmfix <filename>`

specifies a custom empirical mixture of matrices, based on a file containing a set of matrices specified as follows:

```
[<ALPHABET>]
<ncat>

<weight>
<freq1> <freq2> ... <freq20>
```
As in the case of profile mixtures, one should specify on the first line ([<ALPHABET>]) the order in which amino acids are considered. Letters should be separated by spaces or tabs. Then for each component, one should specify, in turn, a weight, an equilibrium frequency profile, and finally, a set of relative exchange rates (separated by space, tabs or returns).

-fxw

With this option, and using one of the pre-specified mixtures C10 to C60, the weights are fixed to the values estimated on the training HSSP database. In all other cases (other pre-specified profile and matrix mixtures, or custom mixtures), the weights are constrained to be all equal. Otherwise, without the -fxw option, the weights are free parameters (endowed with a uniform Dirichlet prior).

5.1.8 Heterotachy

-covarion

Tuffley and Steel’s covarion model (Tuffley and Steel, 1998). This model can be combined with any of the substitution or amino-acid replacement models defined above. However, it does not work in combination with the -ratecat or -cgam options (only with the -dgam or -uni options).

-covext
a slightly different version of Tuffley and Steel’s covarion model, in which the rate of the site is applied both to the substitution and to the switching processes. Under the classical covarion model (-covarion), fast sites make more substitutions when in the on state, compared to slow sites. However, all sites switch between the on and off states at the same rate. In contrast, under the covext model, fast sites also switch faster between on and off. In practice, this means that the rate is simply applied as a multiplier in front of the tensor product matrix. The advantage of doing this is that it can be combined with any model of rate variation across sites.

-mmb1

This is a Bayesian non-parametric version of the mixture of branch lengths (MBL) model (Kolaczkowski and Thornton, 2008; Zhou et al., 2007). Basically a Dirichlet process of vectors of branch length multipliers. These multipliers are drawn from a gamma distribution, whose inverse variance (Φ) is itself considered as a free parameter. Convergence is challenging under this model.

-gbl <partition>

separate (partition) model, with each gene having its own set of branch lengths, which are obtained by multiplying the global branch lengths by gene-specific branch length multipliers. These multipliers are drawn from a gamma distribution, whose inverse variance (Φ) is itself considered as a free parameter. Convergence in the space of topology is challenging under this model. The partition file should be specified as follows:

<G>
nsite\_1 nsite\_2 ... nsite\_G

where G is the total number of genes, and nsite\_g is the number of aligned positions for gene g.

5.1.9 Relaxed clock models

-c1

strict molecular clock

-ln

log normal (Brownian) autocorrelated clock (Thorne et al., 1998)
CIR process (Lepage et al., 2007)

white noise process (Lepage et al., 2007)

uncorrelated gamma multipliers (Drummond et al., 2006).

5.1.10 Prior on divergence times

- **rp** <mean> <stdev>

impose a gamma prior with specified mean and standard deviation for the age of the root.

- **unitime**

uniform prior on divergence times (default option).

- **bd** [<p1> <p2>]

birth death prior on divergence times. The birth death process (with species sampling) has three parameters: the birth rate (\(\lambda\)), the death rate (\(\mu\)) and the sampling fraction (\(\omega\)). Only two of these three parameters are identifiable, which we define as \(p_1 = \lambda - \mu\) and \(p_2 = \lambda \omega\). The dimension of \(p_1\) and \(p_2\) is the inverse of a time. Thus, if fossil calibrations are in Myr, \(p_1\) is the net rate of diversification (speciation - extinction) per Myr. If two real numbers are given, \(p_1\) and \(p_2\) will be fixed to the specified values. Otherwise, they will be considered as free parameters.

- **bdhyperprior** [<mean1> <stdev1> <mean2> <stdev2>]

birth death hyperprior. The two parameters \(p_1\) and \(p_2\) will be endowed with gamma distributions of means and priors specified by the command. Default values are \(10^{-3}\) for all four hyperparameters (i.e. exponential distributions of mean \(10^{-3}\) for both \(p_1\) and \(p_2\)).

- **dir** [<chi>]

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Dirichlet prior. The hyperparameter $\chi > 0$ is such that smaller value of $\chi$ result in a larger variance of the waiting times between successive speciations ($\chi = 1$ reduces to the uniform prior). This hyperparameter can be fixed to a pre-specified value (e.g. $-\chi 0.5$). If no value is given, it will be considered as unknown. Analyses with calibrations require $\chi$ to be specified.

$-\text{cal} \ <\text{calibrations}>$

impose a set of calibrations (see Format section).

$-\text{sb}$

activates the soft bounds option. Soft bounds are implemented as in Yang and Rannala (2006). By default, a proportion of 0.05 of the total probability mass is allocated outside the specified bound(s) (which means, 5% on one side, in the cases of the pure lower and pure upper bounds, and 2.5 % on each side in the case of a combination of lower and upper bound). This default can be overridden as follows: $-\text{sb} \ [<\text{cutoff}>]$.

$-\text{lb} \ [<\text{p} \ <\text{c}>]$  

activates the Cauchy lower bound, such as introduced by Inoue et al. (2010). The $p$ and $c$ parameters (of the $L(t,p,c)$ function in Paml4.2, see Inoue et al. (2010) and the manual of Paml4.2, p 44, equation 1 for details) are by default: $c = 1$ and $p = 0.1$. Note that PhyloBayes does not allow for node-specific cutoff, $p$ and $c$: they will be the same for all calibrated nodes.

5.1.11 Data recoding

Data can be recoded by using the $-\text{recode}$ command:

$-\text{recode} \ <\text{recoding_scheme}>$

Three pre-specified recoding schemes are available:

$\text{dayhoff6} \ (A,G,P,S,T) \ (D,E,N,Q) \ (H,K,R) \ (F,Y,W) \ (I,L,M,V) \ (C)$  
$\text{dayhoff4} \ (A,G,P,S,T) \ (D,E,N,Q) \ (H,K,R) \ (F,Y,W,I,L,M,V) \ (C= ?)$  

Arbitrary recodings can also be specified, by setting $<\text{recoding_scheme}>$ equal to the name of a file containing a translation table formatted as follows:
It is possible to propose that a given amino-acid be considered as missing data, by recoding it as ? (as is done for cysteine in the case of the dayhoff4 recoding).

### 5.2 bpcomp

-\x <burn-in> [every <until>]

Defines the burn-in, the sub-sampling frequency, and the size of the samples of trees to be taken from the chains under comparison. By default, \(<burn-in> = 0, <every> = 1\) and \(<until>\) is equal to the size of the chain. Thus, for instance:

-\x 1000

defines a burn-in of 1000,

-\x 1000 10

a burn-in of 1000, taking one every 10 trees, up to the end of each chain, and

-\x 1000 10 11000

a burn-in of 1000, taking one every 10 trees, up to the 11 000th point of the chains (or less, if the chains are shorter). If the chain is long enough, this implies a sample size of 1000.

-\o <basename>

outputs the results of the comparison in files with the specified base name combined with several extensions:

- \(<basename>.bpcomp\): summary of the comparison;

- \(<basename>.bplist\): tabulated list of bipartitions (splits) sorted by decreasing discrepancy between the chains;

- \(<basename>.con.tre\): consensus tree based on the merged bipartition list.

-\c <cutoff>

tunes the cutoff for the majority rule consensus (posterior probability support under which nodes are collapsed in the final consensus tree). By default, the cutoff is equal to 0.5.
5.3  readpb

-x <burn-in> [every] [until]

Defines the burn-in, the sub-sampling frequency, and the size of the samples of trees to be taken from the chains under comparison. By default, <burn-in> = 0, <every> = 1 and <until> is equal to the size of the chain (as for bpcomp, see above).

-c <cutoff>

sets the cutoff for the majority rule consensus. By default, the cutoff is equal to 0.5 (see also bpcomp)

-r

computes the mean posterior rate at each site.

-rr

computes the mean posterior relative exchangeabilities (only if those are free parameters of the model).

-ss

computes the mean posterior site-specific state equilibrium frequencies.

-m

outputs a histogram approximating the posterior distribution of the number of occupied components of the Dirichlet process mixture of profiles.

5.4  readdiv

readdiv -x <burn-in> [every] [until] [other options] chainname

This program works like readpb. It will output several files:

<chainname>_sample.dates
<chainname>_sample.chronogram
<chainname>_sample.labels
defines the burn-in, the sub-sampling frequency, and the size of the samples of trees to be taken from the chain.

defines the burn-in, the sub-sampling frequency, and the size of the samples of trees to be taken from the chain.

verbose output: outputs all dated trees of the sample into a file.

5.5 ppred

Simple posterior predictive checks  Posterior predictive model checking is often used in Bayesian inference (Meng, 1994; Gelman et al., 2004). It has been introduced in Bayesian phylogenetics by Bollback (2002). The idea is to compare the value of a given summary statistic $S$, such as observed on the true dataset ($s_{obs}$) with the null distribution of $S$ under the model. Posterior predictive checking can be seen as the Bayesian version of parametric bootstrapping, in which the model is first fitted to the data by maximum likelihood, and then replicates of the original dataset are simulated under this maximum likelihood parameter estimate. The collection of values of $S$ realized over this series of simulations is then seen as our sample from the null distribution. Posterior predictive checking works similarly, except that each replicate of the dataset is simulated under a distinct parameter configuration, which is drawn from the posterior distribution. Thus, unlike parametric boostrapping, the posterior predictive approach integrates the uncertainty about the parameter in its null distribution.

In PhyloBayes, posterior predictive checking is implemented using the ppred program. First, a MCMC should be run on the dataset under the model of interest, using pb. Then, using ppred, parameter configurations sampled by this MCMC chain (after burnin) are used to simulate replicates of the original dataset. The summary statistic $S$ is then evaluated on each of these replicates, giving a sample from the posterior predictive null distribution. The deviation of $s_{obs}$ from this distribution can be quantified in terms of the mean deviation, the z-score and a tail area probability (the so-called posterior predictive p-value). In particular, if $s_{obs}$ is far in the tail of the distribution, this indicates the presence of model violations – $S$ is not correctly explained by the model.

In practice, once a chain has been obtained using pb, the following command can be used:

```
ppred -x <burn-in> [<every> <until>] [additional options] <chainname>
```
Without any of the additional options, this command will produce a series of simulated replicates of the original dataset, each of which is then written into a separate file. Alternatively, \texttt{ppred} implements a small number of predefined summary statistics, which are accessible by adding the relevant option to the command shown above. When one of these options is used, the simulation replicates are not written into files. Instead, \texttt{ppred} directly computes the test statistics on the observed and the simulated datasets and then outputs the mean deviation, the z-score and the posterior predictive p-value for the test. These pre-defined options are now described:

\texttt{\textsc{-div}}

Biochemical specificity. Under this option, the test statistic is the mean number of distinct amino acids observed at each column (the mean is taken over all the columns). This test statistic measures how well the site-specific biochemical patterns are accounted for by a model (Lartillot et al., 2007). Often, standards models such as WAG, JTT or GTR are rejected by this statistic, except CAT or CAT-GTR, which are usually not, or weakly, rejected.

\texttt{\textsc{-comp}}

Compositional homogeneity. This test statistic measures the compositional deviation of each taxon. The deviation is measured as the sum over the 20 amino-acids of the absolute differences between the taxon-specific and global empirical frequencies. The program also uses a global statistic, which is the maximum deviation over the taxa.

\textbf{Mapping-based posterior predictive checks} Sometimes, the summary statistic of interest is not just a function of the observed data, but depends on additional features that have to be imputed. In particular, in the context of phylogenetic analysis, one may often be interested in statistics that would capture specific features of the complete substitution history over the tree. This substitution mapping is not observed, although it can be inferred based on the model itself, using the idea of stochastic mapping (Nielsen, 2002).

This idea leads to a slightly different version of the posterior predictive checking formalism, which integrates an additional level of data augmentation. Specifically, for each parameter configuration drawn from the posterior distribution by MCMC (i.e. each point of the MCMC run), two stochastic substitution mappings are performed: one conditional on the true data at the leaves (‘observed’ mapping), and one unconstrained (‘predicted’ mapping). The statistic of interest is then measured on both mappings, and the procedure
is repeated for each point of the run. The observed and the predicted distributions of the statistic can be summarised by their means and variances. The posterior predictive p-value (which is now defined as the number of times that the statistic calculated on the ‘observed’ mapping is more extreme than the value calculated on the ‘predicted’ mapping for the same parameter configuration) is also indicated.

This generalized version of posterior predictive checking is implemented in its full generality by the ancestral program introduced below. It is also implemented by ppred, using the -sat option, although only for one particular summary statistic, aiming at capturing mutational saturation. In phylogenetics, saturation refers to the problem of multiple substitutions at a given site. Such multiple substitutions blur the phylogenetic signal. In addition, they create spurious convergences between unrelated taxa. Under challenging conditions (poor taxonomic sampling, or fast evolving species), these convergences can be mistaken for true phylogenetic signal, thereby creating systematic errors. A good statistical test, to see whether a model is likely to produce artifacts, is therefore to measure how well the model anticipates sequence saturation. If the model does not anticipate saturation correctly, then, it will tend to interpret spurious convergences as true phylogenetic signal, and as a consequence, will more likely create artifacts (Lartillot et al., 2007). The saturation index is defined as the number of homoplasies (convergences and reversions) per sites. This index is a function of the complete substitution history (and not just of the observed data), which is why the extended posterior predictive formalism is needed in that case.

5.6 ancestral

ancestral -x <burn-in> [<every> <until>] [additional options] name

Depending on the exact options, the ancestral program can either reconstruct ancestral sequences, or simulate complete substitution histories. By default, ancestral will reconstruct the ancestral sequences. More precisely, it will estimate the posterior probability of each possible character state, at each node, and for each position of the alignment. It will then output those probabilities in a series of separate files, one for each node of the tree. The format is as follows:

<Nsite> <Nstate> [ALPHABET]
<site_index><prob_1> <prob_2> ... <prob_Nstate>
...

On the first line, the total number of sites of the alignment is indicated, followed by the number of states (e.g. 20 for protein data) and the corresponding alphabet. Then, the
file contains one line for each site, each line displaying the index of the corresponding site followed by the posterior probabilities of each possible character state, in the order specified by the first line. The file itself is named as follows:

<chainename>_sample_<node_number>_<taxon1>_<taxon2>.ancstateprob

Concerning node numbers, a tree in which each node is labelled with its number is also provided as a separate file named <chainename>_sample.labels. For convenience, for each node, two taxa are also specified, of which the target node is the last common ancestor (one should be careful about how the tree is rooted here). Note that files are also produced for leaf nodes. This can be informative in the case where there are missing data: missing entries are then simply imputed based on the posterior probabilities of alternative character states.

When used with the -sub option, ancestral will sample substitution histories from the posterior distribution. The substitution mappings are written into one single (large) file, using an extension of the newick format. Here is an example of a substitution mapping for a subtree of 3 taxa, Tax1, Tax2 and Tax3:

```
((Tax1_I:0.4:I:0.6:M:0.5:L,Tax2_L:0.7:L)L:0.6:L,Tax3_L:1.8:L)L;
```

The substitution mapping is such that the ancestor is in state L, and 2 substitutions have occurred along the branch leading to Tax1, successively, a transition from L to M, and a transition from M to I:

```
|------------------ (L) Tax3
| (L)
| |------- (L) Tax2
|-----|--(L)
|------/------/---- (I) Tax1
L->M   M->I
```

The file <chainename>_sample.sub produced by the program contains one such tree for each site (preceded by the index of the site), and this is repeated for each point of the sample.

point 1
site_1 substitution_tree
site_2 substitution_tree
...
site_N substitution_tree

point 2
Several independent substitution histories can be sampled for each point, using the `-nrep <nrep>` option. Substitution histories sampled from the posterior predictive distribution, in addition to those sampled from the posterior distribution, can also be obtained using the `-ppred` option. Finally, using the `-i` option allows restricting the stochastic mapping to a subset of sites (an interval). Thus, for instance,

```
ancestral -x <burn-in> [<every> <until>] -sub -nrep 2 -i 3 5 name
```

will produce a file with the following structure:

```
point 1
rep 1
posterior
site_3 post_substitution_tree
site_4 post_substitution_tree
site_5 post_substitution_tree
posterior predictive
site_3 ppred_substitution_tree
site_4 ppred_substitution_tree
site_5 ppred_substitution_tree

rep2
posterior
site_3 post_substitution_tree
site_4 post_substitution_tree
site_5 post_substitution_tree
posterior predictive
site_3 ppred_substitution_tree
site_4 ppred_substitution_tree
site_5 ppred_substitution_tree
```

point 2

...
Cross-Validation

Cross-validation (CV) is a general method for evaluating the fit of alternative models. The rationale is as follows: the dataset is randomly split into two (possibly unequal) parts, the training (or learning) set and the test set. The parameters of the model are estimated on the learning set (i.e. the model is ‘trained’ on this subset of empirical observations), and these parameter values are then used to compute the likelihood of the test set (which measures how well the test set is ‘predicted’ by the model). The overall procedure has to be repeated (and the resulting log likelihood scores averaged) over several random splits.

Mathematically, if the original dataset $D$ is split into $D_1$ and $D_2$, where $D_1$ is the training set used for running the chain and $D_2$ the test set, and $M$ is the model under which the chain was run, then the cross-validation score is defined as:

$$\ln p(D_2 \mid D_1, M) = \ln \int p(D_2 \mid \theta, M) p(\theta \mid D_1) d\theta$$

where $\theta$ is the set of (global) parameters of the model. The cross-validation score is a measure of how well the model ‘predicts’ site patterns of $D_2$ after it has ‘learnt’ its parameters on $D_1$. This measure can be computed for alternative models $M_i, i = 1..K$, and models with higher score should in principle be preferred.

Note that this measure automatically takes into account dimensionality issues and will not intrinsically favor models that have more parameters. Intuitively, overfit means that a model focusses too much on irrelevant (random) features of the training set. By definition, these random features will not be consistently reproduced in the test set, and thus, an overfitted model will typically show less good performance once evaluated on the test dataset.

Cross validation needs to be replicated (cross-validation scores typically have a large variability, depending on the exact columns that have been included in $D_1$ and $D_2$). If the original data set is $D$, then random pairs $D_1$ and $D_2$ have to be produced, by randomly sampling columns of $D$ (without replacement), running pb separately on each replicate of $D_1$, and then calculating the cross-validation score on on each resulting chain (and all this for each model $M_i$). The cross-validation score can then be averaged over the replicates for a given model. Then, supposing that a model $M_1$ has a higher average score than $M_2$, the number of replicates for which the score of $M_1$ is indeed higher than the score of $M_2$ can be considered as a measure of the ‘significance’ of this preference for $M_1$ over $M_2$.

Typically, 10-fold cross-validation (such that $D_2$ represents 10% and $D_1$ 90% of the original dataset) has been used (e.g. Philippe et al., 2011), and ten replicates have been run.
(although ideally, 100 replicates would certainly be more adequate). However, alternative schemes are possible. In particular, for faster computation in the case of very large datasets, cross-validation schemes in which the size of $D_1$ and $D_2$ combined together is smaller than the size of $D$ could be useful (as long as $D_1$ is large enough for the parameters to be correctly estimated).

CV is computationally intensive. However, it can be easily parallelized, as each replicated random split can be processed independently. To achieve an optimal parallel processing, a series of distinct programs are proposed in PhyloBayes, corresponding to each step of the method:

- **cvrep**: prepare the replicates;
- **pb**: run each model under each replicated learning set;
- **readcv**: compute the cross-validation scores on each replicate;
- **sumcv**: pool the cv-scores and combine them into a global scoring of the models.

First, to prepare the replicated splits:

```
cvrep -nrep 10 -nfold 10 -d brpo.ali cvb
```

The number of replicates is specified with the **-nrep 10** command, while **-nfold 10** means that we want to perform 10-fold cross validation: that is, for each replicate, the training set will amount to 9/10, and the test set for 1/10, of the initial dataset. The replicates are saved under files using the specified prefix (here, **cvb**):

```
  cvb0_learn.ali, cvb0_test.ali,
cvb1_learn.ali, cvb1_test.ali,
...  
```

Next, a MCMC is run under each learning set, and for each model, using **pb**. To save CPU time, one possibility is to perform these runs under a fixed topology. Although this ignores part of the uncertainty, for relatively large datasets, and for comparing models that usually have large differences in their score, this should not make a big difference.

For instance, suppose we want to compare WAG and CAT:

```
pb -d cvb0_learn.ali -T brpo.tree -x 19 1100 CATcvb0_learn.ali
pb -d cvb0_learn.ali -T brpo.tree -x 1 1100 -wag WAGcvb0_learn.ali
```
The same should be done for the other 9 learning sets. Note that we have subsampled for the CAT model \((-x 10 11000)\) since this model has a more complex parameter configuration, compared to WAG.

It is important to define the name of the chains as shown above, by prefixing the name of the dataset with some prefix standing for the model (e.g. CATcvb0_learn.ali). This is necessary for further processing of the resulting chains.

Once the MCMC runs have proceeded for all the replicates and all the models, the cross-validated likelihoods are calculated using `readcv`:

```
readcv -nrep 10 -x 100 1 CAT cvb
readcv -nrep 10 -x 100 1 WAG cvb
```

The cross-validation score, for each replicate, is the likelihood under the test set, averaged over the posterior distribution of the training set (see integral equation above). Here, it is approximated by averaging over the parameter values sampled by the MCMC when conditioned on the training set, discarding a burnin of 100 points, and taking every point thereafter. The logarithm of the resulting average is stored into a file with a `.cv` extension. There is one such file for each replicate (thus, `cvb0.cv`, `cvb1.cv`, ..., `cvb9.cv`).

Note that, when used with the `-nrep` option such as above, `readcv` will process each replicate successively, which may take a very long time. Alternatively `readcv` can be called on individual replicates. For instance:

```
readcv -rep 2 -x 100 10 CAT cvb
```

will process only the replicate number 2. This makes it possible to split the calculation across several processors.

Once all the cross-likelihood scores, for all the models and all the replicates, have been obtained, they can be collected using `sumcv`:

```
sumcv -nrep 10 WAG CAT cvb
```

This command can be used for more than 2 models (provided that the mean cv-likelihoods are available, and are all stored into the relevant files, with the `.cv` extension):

```
sumcv -nrep 10 WAG CAT GTR cvb
```

This command will average the cv-log-likelihood scores over replicates, using the first model of the list (here WAG) as the reference.

Note that, for computational reasons, cross-validation cannot be done under the continuous gamma or the Dirichlet process models of rates across sites (it works only under the `-dgam` and `uni` models).
7 Priors

Branch lengths ($l$)

- $-\text{lexp}$: $l \sim \text{iid Exponential of mean } \mu$
- $-\text{lgam}$: $l \sim \text{iid Gamma of mean } \mu \text{ and variance } \epsilon \mu^2$ (equivalent to the exponential distribution when $\epsilon = 1$).
- $\mu \sim \text{Exponential of mean } 0.1$
- $\epsilon \sim \text{Exponential of mean } 1$

Rates across sites ($r$)

- $-\text{cgam}$: $r \sim \text{iid Gamma of mean } 1 \text{ and variance } 1/\alpha$
- $-\text{dgam <ncat>}$: $r \sim \text{iid discretized Gamma of mean } 1 \text{ and variance } 1/\alpha$
- $-\text{ratecat}$: $r \sim \text{Dirichlet process of granularity } \zeta \text{ and base distribution Gamma of mean } 1 \text{ and variance } 1/\alpha$
- $\alpha \sim \text{Exponential of mean } 1$
- $\zeta \sim \text{Exponential of mean } 1$

Equilibrium frequency profiles across sites ($\pi$)

- $-\text{ncat } 1$: $\pi \sim \text{Uniform}$
- $-\text{cat}$: $\pi \sim \text{Dirichlet process of granularity } \eta \text{ and base distribution } G_0$
- $-\text{statflat}$: $G_0 = \text{Uniform (on the simplex)}$
- $-\text{statfree}$: $G_0 = \text{generalized Dirichlet, of center } \pi_0 \text{ and concentration } \delta$
- $\pi_0 \sim \text{Uniform}$
- $\delta \sim \text{Exponential of mean } S$ (where $S$ is the number of states, i.e. 4 for nucleotides, 20 for amino acids)
- $\eta \sim \text{Exponential of mean } 10$. 

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Relative exchange rates (rr)

- \( rr \sim \text{iid Exponential of mean 1} \)

Relaxed clock; rates cross branches (\( \rho \))

Rates across branches are relative (i.e. dimensionless). They are multiplied by a global scaling factor, \( \nu \) (expressed in number of substitutions per site and per unit of relative time). See Lepage et al. (2007) for more details.

- \(- \ln: \rho \sim \text{lognormal autocorrelated process of variance parameter } \sigma^2 \) (? in Thorne et al, 1998)
- \(- \text{cir: } \rho \sim \text{CIR process of variance parameter } \sigma^2 \text{ and autocorrelation parameter } \theta \)
- \(- \text{ugam: } \rho \sim \text{iid gamma of variance parameter } \sigma^2 \)
- \( \nu \sim \text{Exponential of mean 1} \)
- \( \sigma^2 \sim \text{Exponential of mean 1} \)
- \( \theta \sim \text{Exponential of mean 1 with the additional constraint that } \theta > \sigma^2/2 \) (and not \( \theta > \sigma/2 \) as written in Lepage et al 2007)

Divergence dates

Divergence dates are relative (dimensionless): root has divergence 1, and leaves divergence 0. These relative dates are then multiplied by a global scale factor \( \text{Scale} \) (expressed in millions of years) which is tuned by the \(-\text{rp} \) (root prior) option.

- \(- \text{unitime: } t \sim \text{Uniform} \)
- \(- \text{dir: } t \sim \text{Dirichlet prior of concentration parameter } \chi \) (when \( \chi = 1 \), prior is uniform).
- \( \chi \sim \text{Exponential of mean 1} \)
- \(- \text{bd: } t \sim \text{birth death process of parameters } \lambda \text{ (birth rate), } \mu \text{ (death rate) and } \omega \text{ (sampling fraction). To ensure identifiability, we set } p_1 = \lambda - \mu \text{ and } p_2 = \lambda \omega. \)
- \( p_1 \sim \text{Exponential of mean 1} \)
- \( p_2 \sim \text{Exponential of mean 1} \)
• \( \text{Scale} \sim \text{Uniform by default. this is an improper prior. Override if possible.} \)

• \(-\text{rp} \ <\text{mean}> \ <\text{stdev}>\): \( \text{Scale} \sim \text{Gamma of specified mean and standard deviation.} \)

**Covarion model**

The covarion parameters can be expressed as

- \( s_{01} = \xi (1 - p_0) \): rate from OFF to ON
- \( s_{10} = \xi p_0 \): rate from ON to OFF

where \( \xi \) is the rate of the ON-OFF process, and \( p_0 \) the stationary probability of the OFF state.

- \( \xi \sim \text{Exponential of mean 1} \)
- \( p_0 \sim \text{Uniform over [0, 1]} \)
References


