PhyloBayes 3.3

a Bayesian software for phylogenetic reconstruction and molecular dating using mixture models

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1. Introduction

PhyloBayes is a Bayesian Monte Carlo Markov Chain (MCMC) sampler for phylogenetic reconstruction and molecular dating using protein and nucleic acid alignments. Compared to other phylogenetic MCMC samplers, the main distinguishing feature of PhyloBayes is the use of non-parametric methods for modelling site-specific features of sequence evolution.

PhyloBayes 3.3 implements a wide spectrum of probabilistic models:
- non-parametric models of site-specific rates, of equilibrium frequency profiles, or even, of the entire GTR matrix. All this using a combination of independent Dirichlet processes.
- Tuffley and Steel's covarion model, as well as the mixture of branch length (mbl) models.
- empirical mixtures of profiles, or of matrices.
- auto-correlated and non auto-correlated models of clock relaxation, meant for molecular dating

Modelling site-specific effects using non-parametric methods

In general, each position of a gene is under a very specific selective constraint. This fact has fundamental consequences for phylogenetic reconstruction. For instance, more constrained sites are slower, and less constrained sites are faster. Similarly, only a subset of the possible character states (4 nucleic acids or 20 amino-acids) is likely to be accepted at a given position during evolutionary times.

This suggests that the overall rate of substitution, the equilibrium frequency profile (more briefly called profile in the following) of the substitution process, or even the entire substitution matrix, should be modeled as site-specific random variables. As has been shown in several previous works, accounting for such variations across sites is indeed crucial, both to obtain a better statistical fit and to alleviate phylogenetic artifacts, due to long branch attraction phenomena.

There are two ways site-specific evolutionary propensities can be modeled. First, one can use a parametric model. In such models, the parametric form of the law describing the distribution of the site-specific feature under investigation is assumed known, up to a few parameters that will be estimated from the data. The best example is the use of a Gamma distribution for modeling the distribution of relative rates of substitution across sites (Yang, 1994).

Alternatively, one can use non-parametric models: the general idea of non-parametric model is that the overall shape of the distribution across sites is not specified a priori, but is itself directly inferred from the data. Thus, it is more general than the parametric method. On the other hand, it requires more data (since more is learnt from them).

There are different non-parametric methods. A practical approach, often adopted in Bayesian inference, is the use of Dirichlet process mixtures. Formally, these are infinite mixtures (mixtures with an countably infinite number of components).

PhyloBayes uses Dirichlet processes at several levels: one for modeling the site-specific rates (Huelsenbeck and Suchard, 2007), and another one aimed at describing the sites-specific profiles (Lartillot and Philippe, 2004). Each site is thus given a rate and/or a frequency vector profile over the 20 amino-acids or the 4 bases. These are combined with a globally defined set of exchange rates, so as
to yield site-specific substitution processes. The global exchange rates can be fixed to flat values (the CAT-Poisson, or more simply, CAT, settings), or inferred from the data (CAT-GTR settings).

In the QMM (‘Q-Matrix Mixture’) model, a Dirichlet process is not defined only on the equilibrium frequency profile, but on the entire rate matrix (Q-matrix): in that case, and contrarily to what happens in the CAT-GTR model, sites do not share the same exchange rates. However, the QMM model seems in fact to be less fit (and computationally more challenging) than the simpler CAT-GTR version, with global exchange rates shared by all sites.

**Empirical mixture models**

The non-parametric models introduced above are flexible. They automatically estimate the distribution of site-specific effects underlying each dataset. But on the other hand, they may require a fairly large amount of data for correct estimation.

An alternative that would be suitable for smaller alignments is offered by the so-called empirical mixture models. Unlike non-parametric mixtures, empirical models have a fixed, pre-determined, set of components, which have been estimated on a large database of multiple sequence alignments. Classical empirical matrices, such as JTT, WAG, and the more recent LG (Le and Gascuel, 2007) are a specific case of empirical model, with only one component. Recently, empirical mixtures have been proposed: either mixtures of profiles (Le et al 2008, Wang et al 2008), or mixtures of 2 or 3 matrices (Le et al, 2008b). All these models are implemented in the current version of PhyloBayes. The user can also specify its own custom set of exchange rates, mixture of profiles, or mixture of matrices.

**Phylogenetic reconstruction and molecular dating**

Once a sample is obtained, it can be marginalized over the parameters of interest. Most often, people are interested in the phylogenetic tree itself: here, PhyloBayes works like usual Bayesian phylogenetic reconstruction programs, and outputs a majority rule posterior consensus tree. But conversely, you might be interested in the site-specific biochemical specificities that have been captured by CAT, or some other site-specific or mixture model. In that case, you can read out the mean posterior site-specific rates and profiles, which are also directly available from the MCMC output. Note that, in contrast to usual sequence profiles, these site-specific features are corrected for the phylogenetic correlations between the sequences. PhyloBayes can also estimate ancestral sequences, and reconstruct the detailed substitution history along the tree.

You may also be interested in the goodness-of-fit of the model, which can be assessed by Bayes factors (with certain restrictions), cross-validations, or posterior predictive resampling procedures.

Finally, PhyloBayes can be used to perform Bayesian molecular dating using relaxed molecular clocks. Several alternative clock relaxation models are available, including the popular log-normal auto-correlated model of Thorne et al. (1998), the CIR process of Lepage et al (2007). Fossil calibrations can be provided as hard (as in Kishino et al 2001) or as soft (as in Yang and Rannala 2006) constraints.
Choosing the model

It is still difficult to get a general idea of the relative merits of the models, as it depends on the dataset, and on the criterion used to measure the fit. But a few trends are observed, which can be used for establishing general guidelines.

Amino-acid replacement models

- For large datasets (more than 1,000 aligned positions), CAT-GTR is virtually always the model with highest fit among all models implemented in PhyloBayes (at least, as measured by cross-validation; Bayes factor have not yet been routinely estimated for comparing CAT-GTR with other models).

- CAT is less fit than CAT-GTR, but generally better than one-matrix or empirical models on large datasets, although this mainly depends on saturation (one-matrix models perform poorly on saturated data)

- Empirical mixture models (UL3, C60 or WLSR5) may be ideal for single gene datasets.

- Concerning the posterior consensus tree, both CAT and CAT-GTR seem to be significantly more robust against long-branch attraction artifacts (LBA), compared to all other models. In some cases, there are differences between the topologies obtained under CAT or CAT-GTR, and it is not yet clear which of the two models is more accurate. In principle, the flat exchange rates of CAT (which were introduced mostly for the sake of computational efficiency) are clearly a limitation, suggesting that CAT-GTR would generally be a better option. But more work is needed here to get a better understanding of the differences between the two models.

- Concerning computational efficiency, CAT is by far the most efficient settings. For large datasets, CAT probably remains the best option, in particular when there is a high level of saturation. However, comparing the results obtained under CAT, CAT-GTR and GTR (or returned by other programs), whenever possible, is important.

- Other settings, such as QMM (Dirichlet processes of GTR matrices), are probably less useful in practice: those models are computationally challenging, and do not seem to bring any significant improvement anyway.

Nucleotide substitution models

Very few options are implemented for specific nucleic acid data in Phylobayes, which is primarily devoted to amino-acid data. However, the CAT-GTR model is probably a very good generic model also for DNA or RNA data, probably better than CAT (which does not correctly handle differences in transition and transversion rates).

Rates across sites

The Dirichlet process over the rates seems to have a better fit than the more classical discrete gamma distribution with 4 categories (as in Huelsenbeck and Suchard, 2007). Thanks to parameter expansion, this model also requires less RAM, and is faster on a per-cycle basis, than the discrete-gamma model. On the other hand, parameter expansion may sometimes result in MCMC convergence and mixing
difficulties. Thus far, we have not seen many cases where the Dirichlet process over the rates was resulting in a significantly different phylogeny, compared to the discrete gamma model. Thus, using the discrete gamma model by default is probably the safest option. Accordingly, and unlike in previous versions of PhyloBayes3, the discrete gamma distribution is now the default model.

Note also that several models or methods proposed by the program are NOT compatible with Dirichlet processes over the site-specific rates, in particular, the covarion model, and estimation of model fit by cross-validation.
2. Downloading and installing PhyloBayes:

PhyloBayes is freely available. It can be downloaded from [www.phylobayes.org](http://www.phylobayes.org). It is provided as a zipped file `phylobayes3.3.tar.gz`. There are several pre-compiled versions, for mac, linux 64 bits and windows.

Once unzipped, the downloaded file will release a directory, `phylobayes3.3`, containing several subdirectories: sources containing the source files; data, containing the data files; several exe_ directories containing the binary files for mac, linux and windows, and aux, containing auxiliary files (necessary for optional postscript outputs). If you need or wish to recompile, simply go into the sources directory, and use the following command:

```
make
```

This will put new binaries directly into the data directory. Since the CIR clock relaxation model (Lepage et al, 2007) requires functions from the gsl library, to make compiling as simple as possible for everyone, the CIR has been deactivated by default. Thus, to compile as described above, you should not need anything particular pre-installed on your computer. In particular, you do not need to install the gsl library. But on the other hand, you will not be able to use CIR. Conversely, if you want to use the CIR model, you need to activate a compilation in the Makefile (at the very beginning of the file, remove the '#' in front of `USE_GSL=-DGSL`). Of course, you then need gsl to be installed.

The general philosophy of PhyloBayes is a bit different from other programs: rather than having one single program that does everything, through standard input and output streams (i.e. keyboard and screen), several small programs have been written, working more or less like unix commands using input and output files. The detailed model and Monte Carlo settings can be controlled through options, starting with the usual hyphen `-`.

The program files can be put anywhere. For instance, you can copy the relevant binaries (depending on your operating system) into the data directory. Then you can create subdirectories in data, which will contain all your analyses using PhyloBayes (for instance, one distinct sub-directory for each phylogenetic session made of related analyses on the same dataset). Alternatively you can put the programs in `/usr/bin/`, if you want to have access to them from anywhere in your file system.

Finally, for the postscript options to work, first, LateX and dvips have to be installed, and second, a global variable needs to be specified, e.g. by writing the following two lines at the end of the `.bashrc` file which is in your home directory:

```
set PBPATH = .../aux_ps/
export PBPATH
```

The ".../" in front of aux/ should be replaced by the path leading to where exactly you have put the aux directory. Note, however, that all programs output only plain text files by default. Therefore, as long as you do not explicitely ask for postscript outputs (using the `-ps` option), all of them will work even if this path has not been specified.
3. Input Data Format

**Sequences**

The main format recognised by PhyloBayes is a generalisation of the PHYLIP format, formatted as follows:

```
<number_of_taxa> <number_of_sites>
taxon1  sequence1...
taxon2  sequence2...
...
```

Taxon names may contain more than 10 characters. Avoid special characters such as ';',' or ')', which will create problems when parsing trees. The best is to only use letters, digits and '_'.

Sequences can be interrupted by space and tab, but not by return characters. Be sure that the lengths of the sequences are all the same, and identical to the lengths indicated in the header. Sequences can be interleaved, in which case the taxon names may or may not be repeated in each block.

PhyloBayes automatically recognizes DNA, RNA, or Protein alphabets. The following characters will be considered all equivalent to "missing data": '-? $ * X x', as well as the degenerate bases of nucleic acid sequences ('B','D','H','K','M','N','R','S','V','W','Y'), and the 'B' and 'Z' characters for protein sequences. Upper or lower case sequences are both recognised, but the case matters for taxon names.

**Special alphabets:** starting from phylobayes3.3e, it is possible to run the CAT or CAT-GTR models under datasets with arbitrary alphabets, of arbitrary size. The file should then be formatted as follows:

```
#SPECIALALPHABET
<number_of_taxa> <number_of_sites> <ALPHABET>
taxon1  sequence1...
taxon2  sequence2...
...
```

where the alphabet is specified by concatenating all characters, in one single word, e.g.:

```
#SPECIALALPHABET
2 4 ABCDE
taxon1  ACABEDE
taxon2  ACEBBDE
```

**Trees**

In some cases, an initial tree can be provided, or the program can be constrained to sample the posterior distribution of parameters under a specified tree topology, which will the remained fixed throughout the analysis. Trees should be provided in NEWICK format, e.g.:

```
(taxon1,(taxon2,taxon3),(taxon4,taxon5));
```
or

\((\text{taxon1}, (\text{taxon2}, \text{taxon3})), (\text{taxon4}, \text{taxon5})\);\)

The first tree is unrooted, the second is rooted. Both are recognised. Branch lengths can be specified, but will be ignored. Trees should always be followed by a ';', and should not contain spaces or tabs.

Taxon names should correspond to the names specified in the data matrix (case sensitive). Alternatively, you can provide integer labels, in which case the order of the taxa provided by the datafile is considered. In the case where names are used: if some names are present in the tree, but not in the matrix, the corresponding taxa will be pruned out of the tree. That is, the spanning subtree containing all the taxa mentioned in the data matrix will be considered as the input tree. Conversely, if some taxa are present in the data matrix, but not in the input tree, this will generate an error message.

**Outgroups**

For molecular dating, a tree needs to be specified, and its rooting matters. If a rooted tree is given as an input, it is used as such. If an unrooted tree is given, an outgroup has to be specified to the program. For this, the outgroup has to be written into a file according to the following format:

```
<number of taxa>
<taxon1>
<taxon2>
...
```

All the taxa making the outgroup should be specified. They should correspond to a split of the tree.

**Calibrations**

Molecular dating requires fossil calibrations to be specified. The format is as follows:

```
<ncalib>
<taxon1a> <taxon1b> <upper_limit> <lower_limit>
<taxon2a> <taxon2b> <upper_limit> <lower_limit>
...
```

The calibration given on each line will be imposed onto the last common ancestor of the two specified taxa. Upper limits are older, and thus, should always specify larger numbers than lower limits. The only exception is that upper or lower limits can be set equal to -1, in which case no limit is enforced. For example:

```
taxon1 taxon2 -1 50
```

means that the node of the last common ancestor of taxon1 and taxon2 should be older than 50 Million years (My), but otherwise, does not have an upper constraint. Likewise:
taxon1 taxon2 70 -1

only specifies an upper constraint of 70 My, and no lower constraint. And finally:

taxon1 taxon2 70 50

specifies an upper and a lower constraint, thus the interval [50,70] My.
IMPORTANT: in the case of soft bounds, and for a node that has both an upper and a lower constraint, it is important to specify both on the same line, thus as:

taxon1 taxon2 70 50

and not

taxon1 taxon2 -1 50
taxon1 taxon2 70 -1
4. General presentation of the programs:

There are different programs in the package (for details about any of these programs type its name without any argument to get an online help):

- **pb/stoppb**: runs and stops the MCMC sampling proper.
- **readpb**: post-analysis program, that returns the posterior consensus tree, as well as a variety of posterior averages.
- **bpcomp**: evaluates the discrepancy of bipartition frequencies between two or more independent runs, and computes a consensus by pooling the trees of all the runs being compared.
- **tracecomp**: evaluates the discrepancy between two or more independent runs, based on the summary variables provided in the files.
- **ppred**: performs posterior predictive analyses.
- **readdiv**: post-analysis program for relaxed clock analyses.
- **ancestral**: post-analysis program for estimating ancestral sequences and reconstructing substitution histories.
- **bf**: for comparing relaxed clock models by Bayes factor numerical evaluation.
- **cvrep/readcv/sumcv**: for comparing models by cross-validation.
- **tree2ps**: converts a tree in newick format into a postscript file.

In this section, a rapid tour of the programs is proposed. A more detailed explanation of all available options for each program is done in the next section.

**Running a chain (pb / stoppb)**

A run of the main program of PhyloBayes (**pb**) will produce a series of points drawn from the posterior distribution over the parameters of the model. Each point defines a value for all these parameters (which include the tree topology, the branch lengths, the biochemical profiles of the mixture, etc.). The series of points defines a Monte Carlo Markov chain, or more simply, a chain.

Each time you create a new chain, you give it a name. An example is given in the package: the **phylobayes/data/brpo/** directory contains a file named **brpo.ali**, which is an alignment of eukaryotic RNA polymerases. To conduct an inference on this data set, first go into the **brpo** directory. From there, you can create a new chain, of name **rnapol1**:

```
  pb -d brpo.ali rnapol1
```

The **-d** option allows you to specify the dataset. There are many other options for specifying the model (see below), but the default options are fine (at least, for proteins). Before starting, the chain will output a summary of the settings.

It is often more convenient to run the chain in background. You can also make two independent chains in parallel:

```
  pb -d brpo.ali rnapol1 &
  pb -d brpo.ali rnapol2 &
```
A series of files will be produced, whose names all start with the name of the chain, combined with a variety of extensions. The most important are:

- `<name>.treelist`: list of sampled trees.
- `<name>.trace`: the trace file, containing a few relevant statistics (e.g. number of generations, log-likelihood, total length of the tree, number of components in the mixture), which can help you checking how fast the chain equilibrates, and how long you will need to run it. The best is to plot the evolution of a few of these statistics, as a function of time, to check for the absence of trend in the long run.

The chains will run as long as you let them run. You can stop, interrupt, or even kill them at any time, and restart them afterwards. In that case, they will resume from the last point that was saved before the interruption. To soft-stop a chain, just type `stoppb <name>`, e.g.:

```
stoppb rnapol1
```

The chain will finish the current cycle before exiting. To restart an already existing chain: `pb <name>` (be careful not to restart an already running chain). In our example:

```
rb rnapol1 &
```

**Checking for convergence (bpcomp and tracecomp)**

The program saves one point after each cycle. A cycle itself is made of a varying number of generations: specifically, the number of generations (second column of the trace file) is defined in PhyloBayes as the number of elementary topological updates tried during a cycle. This number is not constant across cycles, because PhyloBayes implements recursive 'waves' of topological updates along the tree, whose number of steps might depend on the current topology.

In any case, the absolute number of generations performed by a MCMC sampler is not really a relevant measure of the quality of the resulting sample: update mechanisms for the mixture, the topology, or the hyperparameters, are not really comparable, and their mixing efficiency depends very much on the model, the data, and the implementation. We therefore need other ways of estimating when to stop the run, and how much burn-in to discard.

First, one can monitor the evolution of the log-likelihood and other statistics as a function of time. This will be a good indication of how well the profile-mixture has reached the stationary state. But on the other hand, it will not be sufficient, and furthermore, it lacks an objective basis.

An alternative is to run two (or more) chains in parallel, and compare the posterior distributions obtained under these several independent runs. Two main classes of variables can be considered: the summary variables that are displayed in the trace files, and the bipartition frequencies. The first is done using the `tracecomp` program, and the second using the `bpcomp` program. Both use the same syntax:

```
bpcomp -x 100 2 rnapol1 rnapol2
```

Here, with a burn-in of 100, and taking every 2 trees (-x 100 2), the `bpcomp` program will output the largest (`maxdiff`) and mean (`meandiff`) discrepancy observed across all bipartitions (`maxdiff`).
If in addition you specify an output file:

```
bpcomp -x 100 2 -o rnapol rnapol1 rnapol2
```

it will give you a detailed comparison of the frequency of all the bipartitions observed in the chains being compared, and will output a consensus tree made by pooling all the trees of all the chains.

Some guidelines:

- **maxdiff** < 0.1 : good run.
- **maxdiff** < 0.3 : acceptable: gives a good qualitative picture of the posterior consensus.
- 0.3 < **maxdiff** < 1 : the sample is not big yet enough, but it seems to be on the right track.
- if **maxdiff** = 1, even after 10,000 points, this is really bad: it indicates that at least one of the runs is stuck in a local maximum.

Similarly:

```
tracecomp -x 100 2 rnapol1 rnapol2
```

will produce an output summarizing the discrepancies and the effective sizes estimated for each column of the trace file. The discrepancy is defined as $D=2|m_1 - m_2| / (s_1 + s_2)$, where $m_i$ is the mean and $s_i$ the standard deviation associated with a particular column, and $i$ runs over the chains. The effective size is evaluated using the method of Geyer (1992). The guidelines are:

- maximum discrepancy < 0.1 and minimum effective size > 100 : good run,
- maximum discrepancy < 0.3 and minimum effective size > 50 : acceptable run.

**Automatic stopping rule (pb)**

The program now has an option to run several chains in parallel, regularly probing the convergence, and stopping automatically as soon as
- the bipartition (**maxdiff**) and summary variable discrepancies all become lower than a specified threshold c.
- the effective sizes associated to all summary variables are all larger than a specified minimum size.

The burnin is chosen as 1/5 of the chain's length. Thus:

```
pb -d brpo.ali -nch 2 100 0.3 50 rnapol
```

will run 2 chains in parallel, for a minimum of 500 cycles. Then, every 100 cycles, a **bpcomp** and a **tracecomp** between the two chains are automatically done (with a burnin equal to one fifth of the total length of the chain) and the run stops once all the discrepancies are lower or equal to 0.3 and all effective sizes are larger than 50.

Note however that the two chains are run on the same processor, which is different from the situation above, where the two chains could potentially be run on two different processors. Thus, it will take about twice as much time to accomplish the same number of cycles. On the other hand, the program is devised so that the two chains do not run simultaneously, but are just 'intertwined': that is, chain 1 runs for one cycle, then chain 2 runs for 1 cycle, and then both are saved, and so on. In this way, the two
chains can share most the RAM they need, and do not 'overcrowd' the processor on which they run.

**Post-analysis (readbp)**

Once a chain has been obtained, one usually removes the first few hundreds of points (the so-called burn-in), and use the remaining points to compute averages (posterior consensus, mean site-specific profiles, etc.). All this is done with the `readbp` program. The series of points that is used by readbp to compute averages is called a sample (understand : a (sub-)sample of points taken from a chain).

Following our example :

```
readbp -x 100 10 rnapoll
```

will take one tree every ten, discarding the first 10 trees. It will produce a few additional files, among which :

- `rnapoll_sample.general`, containing a few general statistical averages (mean posterior log likelihood, tree length, alpha parameter, number of modes, etc.)
- `rnapoll_sample.con.tre`: the majority-rule posterior consensus tree
- `rnapoll_sample.bplist`: the list of weighted bipartitions.

Various options allow you to compute several other posterior averages, such as site-specific rates and equilibrium frequency profiles.

**Model evaluation.**

Apart from computing averages from the posterior distribution, Bayesian inference also allows for model comparison and goodness-of-fit analyses. Model comparison often consists in computing Bayes factors. However, numerical evaluation of Bayes factor is a very difficult task. Simple estimators often fail. For instance, the popular harmonic mean estimator is not reliable (Lartillot and Philippe, 2006), and is thus not implemented here. Path sampling, or thermodynamic integration, is more reliable, and we have used it previously for comparing phylogenetic models (Lartillot and Philippe 2006). But it is time-consuming, and is not implemented here for general substitution models. In the present version, we only provide thermodynamic integration for comparing clock models under the normal-approximation: this can be done using the program called `bf`. Note however that Bayes factor computations have not been carefully maintained over the years. It is thus not guaranteed that they will be compatible with all subsequently added options. Please send us a mail if you want to perform such computations.

For example:

```
bf -cov moc10.oes -ln moc10ln
```

will compute the Bayes factor between the log-normal and the deconstrained model. You can do this for several models, and then choose the model returning the largest Bayes factor.

Apart from Bayes factors, another general model comparison method in Bayesian inference is cross-
validation. It is implemented in the PhyloBayes package. The exact procedure is a bit involved, and is explained at the end of this manual.

Finally, models can also be assessed by posterior predictive resampling, which is the Bayesian equivalent of the parametric bootstrap often used in Maximum Likelihood. The idea is to compare the observed value of a given test statistic \( s \) on the true data, with the distribution of \( s \) on data replicates simulated under the reference model. If the observed value of \( s \) deviates significantly, this means that the reference model is rejected.

According to parametric bootstrap, replicates should be simulated under the ML value of the parameter. The posterior predictive formalism is slightly different: simulations should instead be performed using parameter values drawn from the posterior distribution. This has the advantage of ‘smoothing out’ your null distribution with respect to the uncertainty over your parameter estimates. On the other hand, the test may then be sensitive to the prior distribution.

Posterior predictive resampling can be done using the `ppred` program. By default, `ppred` takes a chain as an argument (like `readpb`), and for each point of the sample, simulates a series of replicates of the dataset using the topology and the parameters defined by the current point. For instance, say you have run a chain of length 1100 points, under the wag model:

```
pb -d brpo.ali -x 1 1100 -wag brpowagl
```

Once this chain is done, you can then use it to obtain a series of 100 replicates using `ppred`:

```
ppred -x 100 10 brpowagl
```

Here, the options tell `ppred` to take one point every 10, after a burn-in of 100, and use the parameters defined by each of these points to simulate replicates. Once replicates have been obtained, any test statistic which can be computed on the true dataset (e.g. compositional deviation of a given taxon) can also be computed on each of the replicates, which give a null distribution, with which to compare the true value obtained on the original dataset. The implementation of the test statistic is left to the user, except for 3 pre-defined statistics: saturation, site-specificity, and compositional homogeneity.
5. Divergence times.

PhyloBayes allows for molecular dating under a variety of relaxed clock models and priors on divergence times. There are quite a few options here, and as in the case of tree reconstructions, we tried to provide good defaults. But on the other hand, it is not easy to find universal default settings in case of molecular dating analyses (which are inherently sensitive to prior specifications). There are also several potential issues to be aware of. In the following, we give a brief presentation of the available models and options, while emphasizing the most important things to be careful about.

A brief example:

Molecular dating can be done using pb program. It only works under a fixed topology, that has to be specified using the -T option. The molecular dating option of pb is activated by specifying a model of clock relaxation, or by specifying calibrations. Here is an example (files are in the moc directory):

```
pb -d moc.ali -T moc.tree -r bikont.outgroup -cal calib -ln -rp 2000 2000 mocln1
```

Here, the program is given a protein sequence alignment (-d moc.ali, a subalignment of Douzery et al., 2004), a tree file (-T moc.tree), an outgroup (-r bikont.outgroup, so as to have a basal split between unikonts and bikonts), and a file containing 6 calibrations (-cal calib) specified in Million of years. A log-normal autocorrelated relaxed clock (-ln) is assumed, together with a uniform prior on divergence times (default prior). A gamma prior of mean 2000 and standard deviation 2000 Myrs (thus, an exponential of mean 2000 Myrs) is specified for the age of the root.

The chain can be run for a few thousands of cycles, and the posterior mean chronogram can be obtained using readdiv:

```
../readdiv -x 2500 1 mocln1
```

Here, the first 2500 saved points are excluded. The readdiv program will output a chronogram (<chainname>_sample.chronogram), as well as a file containing the list of divergence times, with standard errors (<chainname>_sample.dates). The labels of the nodes are indicated in a specific tree, in the file with the ".labels" extension.

An important detail here is that the prior that we specified for the divergence times (a uniform prior on the relative ages w.r.t. the root, plus a gamma for the root) is the prior before the calibrations. So, we need to check that the prior on divergence times once the calibrations have been included is satisfactory. This can be done by running a chain under the prior:

```
pb -d moc.ali -T moc.tree -r bikont.outgroup -cal calib -ln -rp 2000 2000 -prior moclnprior1
```

and checking that the resulting distribution over the age of the root, and more generally over the ages of the nodes of interest, is sufficiently wide/uninformative to be used as a prior.
Relaxed clock models:

PhyloBayes provides the following relaxed clock models:

- **cl**: strict molecular clock
- **ln**: log normal model (Thorne et al, 1998)
- **cir**: CIR process (Lepage et al, 2007)
- **wn**: white noise process (Lepage et al, 2007)
- **ugam**: uncorrelated gamma multipliers (Drummond et al, 2006).

See Lepage et al (2007) for a more detailed review of all these models. The autocorrelated lognormal and CIR, and the uncorrelated gamma models are probably the three most important models to try (the strict clock is usually strongly violated, and the white noise is a particular case of the uncorrelated gamma).

Calibrations and time scale:

Calibrations are provided as a separate file (see 'Format' section), and are specified using the following option: `-cal <calibration_file>`

The time scale (the unit of time) is implicitly defined by the calibrations. Thus, if your calibrations are in Million years, 1 time unit = 1 Myr. If no calibration is specified, the total duration from root to tips is 1000 time units.

Priors on the age of the root:

We strongly advise the explicit declaration of a prior for the age of the root. This can be done by specifying a calibration interval in the `<calibration_file>` file. Alternatively;

- **rp** `<mean> <stdev>`

imposes a gamma prior with specified mean and standard deviation. If this option is not used (and if no calibration is specified for the root), then
- under the uniform and the Dirichlet priors, the root age is uniformly distributed (there is an upper limit at 5000). This is admittedly a poor prior.
- under the birth death prior, the marginal distribution implied by the birth death prior is used. It is a proper, but it may be difficult to control, due to its sensitivity to the parameters of the birth death process. In addition, it amounts to interpreting the model perhaps too literally, and assuming that the speciations are indeed described by a birth death process. If, on the other hand, the birth death prior is merely understood as a flexible prior, then there is no reason to assume that it should correctly describe our prior uncertainty concerning the root.

In any case, we also strongly suggest to run the program under the prior (using the -prior option, see below), to experimentally check what the prior on the root looks like in practice.
Priors on divergence time:

Uniform prior

By default, or using the -unitime option, the prior on divergence time is uniform.

Birth death

A good alternative to the uniform is the birth death prior (see Inoue et al, 2010, for a detailed comparison between the two). The birth death prior is attractive, because it is more flexible, and because it offers a somewhat more direct control on the influence of calibrations on the prior. In addition, it makes it possible to implement soft bounds (Yang and Rannala, 2006). Soft bounds are explained below.

The birth death can be specified using the -bd option. The birth death process (with species sampling) has three parameters: the birth rate ($\lambda$), the death rate ($\mu$) and the sampling fraction ($\rho$). However, under the present conditions, only two parameters are identifiable, which we define as (Lepage et al 2007):

\[ p_1 = \lambda - \mu, \]
\[ p_2 = \lambda \rho. \]

The dimension of $p_1$ and $p_2$ is the inverse of a time. The time scale defined as explained above is used here. For instance, if your calibrations are in Myrs, $p_1$ is the net rate of speciation - extinction per Myr (note that, in versions anterior to 3.3, $p_1$ and $p_2$ were expressed in the inverse of the prior mean age of the root, which was less convenient).

By default, $p_1$ and $p_2$ are considered as unknown, and are both given an exponential prior of mean $10^{-3}$. It is possible to specify their value manually, or to tune the mean and the variance of their prior (see detailed options, next section). However, it is not totally clear to us how to determine reasonable values. Ideally, the best solution would probably be to tune $p_1$ and $p_2$ automatically to their Empirical Bayes estimate (i.e. automatically maximize the marginal likelihood of the model w.r.t. $p_1$ and $p_2$). But this is not yet implemented. Thus far, however, we have not seen any clear evidence that the default prior has a strong influence on the posterior estimates (although this might need further investigations).

On the other hand, we do see a strong influence of the choice between the birth death and the uniform prior (as first pointed out by Inoue et al, 2010), and we strongly suggest to make systematic comparative analyses.

Comparing PhyloBayes and Paml

If you want to compare estimations obtained under Paml and PhyloBayes, you need to match the parameters of the birth death and the priors of the relaxed clock between the two programs. Here is a method allowing you to specify the parameters in Paml so as to match as closely as possible the settings in PhyloBayes:

Run PhyloBayes. The readdiv program will give you, along with the posterior mean chronogram, a
posterior mean estimate of $p_1$ and $p_2$, as well as an estimate of the age of the root ($T_{\text{root}}$). These are given on the standard output, and are also stored in a .hyperparam file.

You can translate the estimates of $p_1$ and $p_2$ into values of $\lambda$, $\mu$ and $\rho$, to be used for a Paml analysis. However, there is an indeterminacy here, since we want to specify 3 parameters out of 2. And also, we should be careful about potentially different time scales between Paml and PhyloBayes. Imagine for instance that the time scale is $T_{\text{paml}}=100$ Myrs in Paml, and $T_{\text{pb}}=1$ Myr in PhyloBayes. Then, there are 2 subcases:

- if $p_1 > p_2$, then you can set $\mu=0$. Combined with proper rescaling, this leads to:
  
  $$
  \lambda = T_{\text{paml}} / T_{\text{pb}} * p_1 = 100 \ p_1 \\
  \mu = 0, \\
  \rho = p_2 / p_1.
  $$

- if $p_1 < p_2$, then you can set $\rho=1$, and rescale:

  $$
  \lambda = T_{\text{paml}} / T_{\text{pb}} * p_2 = 100 \ p_2 \\
  \mu = T_{\text{paml}} / T_{\text{pb}} * (p_2 - p_1) = 100 \ (p_2 - p_1) \\
  \rho = 1.
  $$

Another potentially important difference is the prior on $\sigma^2$ and on the overall substitution rate (called $\nu$ in PhyloBayes, see Priors section, and 'gene_rate' in Paml). In PhyloBayes, these priors are exponentials of mean 1, but the unit of $\nu$ and $\sigma^2$ in PhyloBayes is the inverse of the relative time (i.e. the total time from root to tips). Working in relative time is convenient, both for technical reasons related to MCMC mixing, and because this makes the priors less dependent on the particular case under investigation (e.g. the mean number of substitutions per site from the root to the tip of a tree is generally of a few units, and therefore, a prior of mean 1 is not too unreasonable in most cases).

Rescaling the exponential priors on $\sigma^2$ and on $\nu$ leads to exponential priors of mean $T_{\text{root}}/T_{\text{paml}}$ (i.e. of rate $T_{\text{paml}}/T_{\text{root}}$) in Paml. Thus, for instance, if the age of the root is estimated to be at around 200 Myrs by PhyloBayes, and if the time scales are the same as above, then $T_{\text{root}} / T_{\text{paml}} = 0.5$, and you just need to set in Paml:

$$
\text{rgene\_gamma} = 1 \ 0.5 \\
\text{sigma2\_gamma} = 1 \ 0.5
$$

Otherwise, the two programs differ in several model and prior specifications. But normally, under equivalent parameters and substitution process, they should give similar divergence date estimates.

**Dirichlet prior:**

Another prior is also available: `-dir [\chi]` : Dirichlet prior. $\chi > 0$ is the hyperparameter (with smaller value resulting in a larger variance of the waiting times between successive speciations; $\chi=1$ reduces to the uniform prior). $\chi$ can be fixed or be inferred from the data, depending on whether a real number is specified in the command. Note that in the specific case of the Dirichlet prior, analyses with calibrations require $\chi$ to be specified, which makes this prior not really practical. In practice, it seems that the estimated values of $\chi$ are not very different from 1, which means that the Dirichlet prior is not
fundamentally different from the uniform prior anyway.

Cauchy lower bounds:

The Cauchy lower bound has been introduced by Inoue et al, 2010. Use `-lb [<p> <c>]` to activate this option. The p and c parameters (the L(t,p,c) in Paml4.2, see Inoue et al, 2010, and Paml4.2 manual, p 44, equation 1 for details) are by default: c=1 and p=0.1. This can be overriden using the following option: `-lb [<p> <c>]

Note that PhyloBayes does not allow for node-specific cutoff, p and c: they are necessarily the same for all calibrated nodes. Note also that you should NOT provide independently an upper and a lower bound for a given node as 2 separate lines of the input file. You should provide them jointly, on the same line (see Data Input Format). In the current version, failing to do that will generate an error and the program will exit.

Soft and hard constraints:

By default, calibrations define hard constraints. PhyloBayes3 also implements soft bounds (as described in Yang and Rannala, 2006, and as implemented in Paml4.2). The soft bound model requires a birth death prior on divergence times. In PhyloBayes, soft bounds are activated by adding the `-sb` option (which should therefore always be used in combination with `-sb`). Calibrations should be provided exactly in the same way as for the hard bound case. For instance, the following command will activate soft bounds, using the default configurations (which should be fine in most cases):

```
pb -d moc.ali -T moc.tree -r bikont.outgroup -ln -cal calib -sb -rp 2000 2000 name
```

By default, a proportion of 0.025 of the total probability mass is allocated outside the specified bound(s) (which means, 2.5% on one side, in the cases of the pure lower and pure upper bounds, and 2.5% on each side in the case of a combination of lower and upper bound. Note that this option was not correctly described in previous versions of the manual). This default can be overriden as follows: `-sb [<cutoff>]`.

Running the software under the prior:

As a general rule, when doing molecular dating experiments, one should always run the program under the prior, and check that the distributions obtained for the nodes of interest are sufficiently wide (i.e. non informative), before doing any posterior sampling. In particular, it is always a good thing to look at the prior age distribution of the root.

This is due to the fact that the prior proposed by the user on the age of the root (using the `-rp` option, see below) and on the other nodes (uniform, birth death priors) is the prior without calibrations, and can sometimes be very different from the prior conditional on the calibrations. The option `-prior` allows one to do this, by desactivating all likelihood computations (this option works only for molecular dating studies). Note however that, when the number of taxa is large, and the prior is indeed sufficiently wide, it may take a very long time for the MCMC sampler to explore the full range of the prior.
6. Detailed program options.

pb

General options:

-d <datafile>

allows one to specify a file containing aligned sequences. For the exact specifications of the formats accepted by the program, see above (see: Input Data Format).

-dc

constants sites are removed.

-cov <varcovar>

specifies a variance-covariance matrix. This matrix can be a file produced by the estbranches program of Thorne et al.

-t <treefile>

forces the chain to start from specified tree.

-T <treeefile>

constrains the chain to run under the topology specified in <treefile>. It will thus only samples from the posterior distribution over all other parameters (branch lengths, alpha parameter, etc.), conditional on the specified topology.

-r <outgroup>

re-roots the tree, using the specified outgroup, before starting the chain. Current substitution models implemented in PhyloBayes are reversible, so this rerooting is useful only for clock models.

-s

in order to save disk space, by default the current version only saves the trees explored by the chain during MCMC. This is enough if what you want to do is only computing a consensus tree. However, if you wish to do a more detailed analysis, such as looking at the site-specific biochemical profiles, or perform posterior predictive tests, then, you need the program to save the whole parameter vector for each visited point, and not only the current tree. The -s option is meant for this.

Fixed topology analyses (where you have used the -T option to constrain the tree topology) will automatically activate the -s option.
-f

forces the program to overwrite an already existing chain with same name.

-x <every> [<until>]

specifies the saving frequency, and (optional) the number of points after which the chain should stop. If this number is not specified, the chain runs 'forever'. By definition, -x 1 corresponds to the default saving frequency (a cycle is not a well defined concept in MCMC programs: it is very model AND implementation dependent). In some cases, you may realise that your samples are very correlated, and that they are taking too much space on your hard-drive. In that case, it would make sense to save points less frequently, say 10 times less often: then, you should use the -x 10 option.

-nchain <nchain> [<step> <cutoff> <eff_size>]

runs nchain chains in parallel, for a minimum of burnin_factor x step cycles, where burnin_factor is a parameter equal to 5 by default; then, every step cycles, checks (using bpcomp and tracecomp) the discrepancy between the two chains and the effective sample sizes, and stops if this discrepancies are below the specified cutoff, and the effective sample sizes are larger than the specified eff_size.

-b <burnin_factor>

tunes the burnin-factor parameter. This parameter is used to define the burnin when checking for the convergence of independent chains (see -nchain option). It is equal to 5 by default (thus, by default, the burnin is 1/5 of the total length of the chains).

Evolutionary models

Prior on branch lengths:

-lexp: prior on branch lengths is a product of i.i.d. exponential distributions.

-lgam: prior on branch lengths is a product of i.i.d. gamma distributions (this is more flexible than the exponential, as it allows to independently control the mean and the variance of the branch length distribution).

-meanlength <meanlength>: fixes the mean of the exponential (or gamma) priors to the specified value. Without this command, this mean is considered as a free parameter, endowed with an exponential prior of mean 0.1. In the case of the gamma prior, the variance/mean ratio is always considered as a free parameter, with an exponential prior of mean 1.

Rates across sites:

-ratecat: the distribution of rates across sites is emulated by a Dirichlet process (as in Huelsenbeck and Suchard, 2007).
-uni: uniform-rate model.
-
cgam: continuous gamma distribution.
-
dgam [<n>]: discrete gamma distribution with <n> categories.

Important notice: unlike in previous versions of PhyloBayes 3, the default rate model is now the discrete gamma distribution, and not anymore the Dirichlet process. There are several reasons for this. First, for large datasets, it seems that the combination of -cat and -ratecat can sometimes be quite challenging in terms of convergence and mixing. And second, cross-validation does not accept the ratecat model. Apparently, default models that cannot be used for cross-validation has tricked several users into wasting a lot of CPU time running chains that could not be used for the intended purpose.

Relative exchangeabilities (exchange rates):
-
-poi (or -poisson): exchange rates all equal to 1. The model is then a mixture of Poisson (Felsenstein81) processes.
-
-
-
gtr: general time reversible matrix: exchange rates are free parameters, with prior distribution a product of independent exponential distributions of mean 1.
-
-rr <filename>: exchange rates are fixed, according to the specifications provided by the specified file. The file should read as follows:

[<ALPHABET>]
<rr1_2> <rr1_3> ... <rr1_20>
<rr2_3> <rr2_4> ... <rr2_20>
...
<rr18_19> <rr18_20>
<rr19_20>

You have to specify the order in which amino acids should be considered on the first line ([<ALPHABET>]). Letters should be separated by spaces, tabs or returns. Supposing you specify the amino-acids in the alphabetical order of the one letter code (A C D E F G H I K L M N P Q R S T V W Y), the order in which the file will have to specify the relative rates is as follows: between A and C, then between A and D ... between A and Y on the first line; then on the second line, between C and D, etc. Tabulations, spaces and returns are all considered equivalent: only the order in which all these real numbers are specified is important.

Profile mixture:
-
cat: Dirichlet process (number of components, weights and profiles are all inferred from the data).
-ncat <n> : mixture of <n> profiles. (number of components is fixed, profiles and associated weights are inferred).

By default, the Dirichlet process is activated. Fixing the number of components of the mixture (-ncat <n>) yields a poor mixing.

Empirical profile mixture models:

- catfix <predef> : mixture of a set of pre-defined profiles (only the weights are inferred). <predef> can be either one of the following keywords: C20, C30, C40, C50, C60, which correspond to empirical profile mixture models, learnt on the HSSP database (Le et al, 2008); or WLSR5, which correspond to the model of Wang et al, 2008. Note that this latter model actually defines 4 empirical profiles, which are then combined with a fifth component made of the empirical frequencies of the dataset.

- catfix <filename> : where <filename> is the name of a file containing a set of profiles specified as follows:

[[<ALPHABET>]
ncat
<weight> <freq1> <freq2> ... <freq20>
<weight> <freq1> <freq2> ... <freq20>
...

where <ncat> is the number of profiles, and each line following this number should be a set of 21 real numbers, defining a weight, and then a profile of equilibrium frequencies. You should specify the order in which amino acids should be considered on the first line ([<ALPHABET>]). Letters should be separated by spaces, tabs or returns. Tabulations, spaces and returns are all considered equivalent.

Combining profiles and exchange rates:

Any set of exchange rates can be combined with any of the three settings for the mixture. But the SAME set of exchange rates will be used by all components. To have different sets of exchange rates, see below (matrix mixture models)

For instance, -cat -gtr makes an infinite mixture model whose components differ by their equilibrium frequencies, but otherwise share the same set of relative exchange rates (themselves considered as free parameters). As another example, -catfix WLSR5 -jtt defines the Wang et al 2008 model: a model with 5 components, each of which is a matrix made from the relative exchange rates of the JTT matrix, combined with one of the 4 vectors of equilibrium frequencies defined by Wang et al 2008, plus one vector of empirical frequencies.

The default model is -cat -poi, that is, a mixture of Poisson (Felsenstein1981) processes. The program works best under those conditions.

Note also that, if you activate non-Poisson exchange rates, the mixture model is automatically desactivated by default: you only have one such matrix, conditioning all the sites. Thus, the -gtr
option is equivalent to `-gtr -ncat 1`. Thus if you want a Dirichlet process combined with a GTR matrix of exchange rates, you have to EXPLICITELY call it `-gtr -cat`.

**Matrix mixture models:**

- `qmm`: activates the QMM model, which is a mixture of matrices (thus, allowing for DISTINCT sets of exchange rates and equilibrium frequencies for each component of the mixture).

- `qmmfix ul2` or `qmmfix ul3`: defines the empirical mixture with 2 or 3 components (Le et al, 2007b).

- `qmmfix <filename>`: where `filename` is the name of a file containing a set of matrices specified as follows:

```xml
[<ALPHABET>]
<ncat>
<weight>
<freq1> <freq2> ... <freq20>
<rr1_2> <rr1_3>    ...   <rr1_20>
<rr2_3> <rr2_4> ...  <rr2_20>
...  
<rr19_20>
<weight>
<freq1> <freq2> ... <freq20>
<rr1_2> <rr1_3>    ...   <rr1_20>
<rr2_3> <rr2_4> ...  <rr2_20>
...  
<rr19_20>
...  
```

As for profile mixtures, you should specify the order in which amino acids should be considered on the first line (`[<ALPHABET>]`). Letters should be separated by spaces, tabs or returns. Then for each component, you should specify, in turn, a weight, an equilibrium frequency profile, and finally, a set of relative exchange rates. Tabulations, spaces and returns are all considered equivalent: only the order in which all this real numbers are specified is important.

**Weights of mixture components:**

By default, the weights are free parameters (endowed with a uniform Dirichlet prior).

- `fxw`: In case you use one of the pre-specified mixtures C10 to C60, the weights are fixed to the values estimated on the training HSSP database. In all other cases (other pre-specified profile and matrix mixtures, or custom mixtures), the weights are fixed, and set all equal. Thus, this option is interesting only when using empirical profile mixtures on very small datasets.
Heterotachy:

- **covarion**: Tuffley and Steel's covarion model. This can be combined with any of the substitution or amino-acid replacement models defined above. However, it does not work in combination with the -ratecat or -cgam options. Thus, you should either use a more classical discrete gamma distribution with the -dgam option (or no variations or rates across sites, with the -uni option).

- **covext**: a slightly different version of Tuffley and Steel's covarion model, in which the rate of the site is applied both to the substitution and to the switching processes. That is: under the classical covarion model (-covarion), fast sites make more substitutions when in the on state, compared to slow sites. But all sites switch between the on and off states at the same rate. In contrast, under the covext model, fast sites also switch faster between on and off. In practice, this means that the rate is simply applied as a multiplier in front of the tensor product matrix. The advantage of doing this is that it can be combined with any model of rate variation across sites (although you may encounter identifiability problems in practice).

- **mmb1**: This is a Bayesian non-parametric version of the mixture of branch lengths (MBL) model (Kolaczkowzki and Thornton, 2008, Zhou et al 2008). Basically a Dirichlet process of vectors of branch length multipliers. These multipliers are drawn from a gamma distribution, whose inverse variance ($\phi$) is itself considered as a free parameter. Convergence is challenging under this model. You should always check carefully for convergence of the topology and of the continuous parameters.

- **gbl <partition>**: separate model, with each gene having its own set of branch lengths, that are obtained by multiplying the global branch lengths by gene-specific branch length multipliers. These multipliers are drawn from a gamma distribution, whose inverse variance ($\phi$) is itself considered as a free parameter. Convergence in the space of topology is challenging under this model. The partition file should be specified as follows:

  ```
  <G>
  nsite_1 nsite_2 ... nsite_G
  where $G$ is the total number of genes, and $nsite_g$ is the number of aligned positions for gene $g$.
  ```

Relaxed clock models:

- **cl**: strict molecular clock
- **ln**: log normal model (Thorne et al, 1998)
- **cir**: CIR process (Lepage et al, 2007)
- **wn**: white noise process (Lepage et al, 2007)
- **ugam**: uncorrelated gamma multipliers (Drummond et al, 2006).

prior on divergence times

- **rp <mean> <stdev>**: impose a gamma prior with specified mean and standard deviation for the age of the root.

- **unitime**: uniform prior on divergence times (default).

- **bd [p1] <p2>**: birth death prior on divergence times. The two parameters p1 and p2 are the
hyperparameters of the birth death process (p1 = birth rate - death rate, and p2 = birth rate * sampling rate, see Lepage et al 2007 for details). If two real numbers are given, p1 and p2 will be fixed to the specified values. Otherwise, they will be considered as free parameters.

-bdhyperprior [<mean1> <stdev1> <mean2> <stdev2>] : birth death hyperprior. The two parameters p1 and p2 will be endowed with gamma distributions of means and priors specified by the command. Default values are $10^{-3}$ for all four hyperparameters (i.e. exponential distributions of mean $10^{-3}$ for both p1 and p2).

-dir [<chi>] : Dirichlet prior. $\chi > 0$ is the hyperparameter (with smaller value resulting in a larger variance of the waiting times between successive speciations). $\chi$ can be fixed or be inferred from the data, depending on whether a real number is specified in the command. Note that in the specific case of the Dirichlet prior, analyses with calibrations require $\chi$ to be specified.

-cal <calibrations> : impose a set of calibrations see Format section for the exact format.

-sb activates the soft bounds option.

By default, a proportion of 0.05 of the total probability mass is allocated outside the specified bound(s) (which means, 5% on one side, in the cases of the pure lower and pure upper bounds, and 2.5% on each side in the case of a combination of lower and upper bound). This default can be overridden as follows: -sb [<cutoff>].

Another default concerns the p and c parameters of the Cauchy lower bound (the $L(t, p, c)$ in Paml4.2, see Inoue et al. 2010, and Paml4.2 manual, p 44, equation 1 for details). By default, $c=1$ and $p=0.1$, which can be overridden as follows: -lb [<p> <c>].

Note that PhyloBayes does not allow for node-specific cutoff, p and c: they are necessarily the same for all calibrated nodes. Note also that you should NOT provide independently an upper and a lower bound for a given node as 2 separate lines of the input file. You should provide them jointly, on the same line (see Data Input Format).

Running the models under their prior:

-prior: desactivates all likelihood computations. As a result, the program samples from the prior of the model, such as specified by the command. This option works only for molecular dating studies, but it is an important one in this context. In general, the prior proposed by the user on the age of the root (using the -rp option) is the prior without calibrations, and can sometimes be very different from the prior conditional on the calibrations. Therefore, one should always run a MCMC under the prior, and check that the distribution of the root's age thus sampled by the MCMC is sufficiently wide (i.e. non informative), before proceeding with a posterior sampling.

Data recoding

Data recoding is the generalisation of the RY coding sometimes used for nucleotide sequences. It is meant for two things: first, recoding several biochemically similar amino-acids (or bases) as one single
letter eliminates the potentially saturated signal brought about by repeated substitutions among those amino-acids (or bases). Second, it can also be used to remove possible compositional biases (RY coding, or for proteins, the case of Arginine and Lysine, whose relative frequency is very sensitive to the GC content).

Data recoding has not yet been used very often in practice. Theoretically speaking, it is not totally satisfactory: in principle, one would prefer a model that automatically accounts for higher substitution rates between equivalent amino-acids, or accomodates variations of the amino-acid composition along the lineage. In addition, there is no objective way to compare the "fit" of alternative recoding schemes. For instance, comparing cross-validated likelihoods, or computing Bayes factors, between alternative recoding schemes would not make sense. But recoding can still be used as an exploratory device. In practice, it can sometimes yield interesting results (REF), albeit most often at the cost of a significant loss of phylogenetic signal.

You can recode the data by using the -recode command:

```bash
-recode <recoding_scheme>
```

or

```bash
-rec <recoding_scheme>
```

Three recoding schemes are available: dayhoff6, and hp. They are characterised as follows:

- **dayhoff6**
  - (A,G,P,S,T) (D,E,N,Q) (H,K,R) (F,Y,W) (I,L,M,V) (C)

- **dayhoff4**
  - (A,G,P,S,T) (D,E,N,Q) (H,K,R) (F,Y,W,I,L,M,V) (C= ?)

- **hp**

You can also define your own recoding, by setting `<recoding_scheme>` equal to the name of a file containing a translation table formatted as follows:

```
<AA1> <Letter1>
<AA2> <Letter2>
. . .
<AA20> <Letter20>
```

You can propose ‘?’ for a given amino-acid, in which case, it will be simply considered as equivalent to "no data" (as the cysteine, in the case of the dayhoff4 recoding).
**bpcomp**

- `x <burn-in> [every until]`

defines the burn-in, the sub-sampling frequency, and the size of the samples of trees to be taken from the chains under comparison. By default, `<burn-in> = 0, <every> = 1` and `<until>` is equal to the size of the chain. Thus, for instance:

```
x 100
```

defines a burn-in of 100

```
x 100 10
```
a burn-in of 100, taking one every 10 trees, up to the end of each chain, and

```
x 100 10 1000
```
a burn-in of 100, taking one every 10 trees, up to the 1000th point of the chains (or less, if the chains are shorter). If the chain is long enough, this implies a sample size of 90.

- `o <basename>`

outputs the results of the comparison in files with name `<basename>`, combined with several extensions:

- `<basename>.bpcomp` : summary of the comparison
- `<basename>.bplist` : tabulated list of bipartitions (splits), sorted by decreasing discrepancy between the chains.
- `<basename>.con.tre` : consensus tree based on the merged bipartition list.

- `ps`

activates the postscript output (this requires LateX to be installed on your computer).

- `v`

make a 'verbose' account of the bipartitions in the `.bplist` file. This allows one to more easily spot which groups of taxa display discrepancies in their supports obtained across the runs.

- `c <cutoff>`

tunes the cutoff for the majority rule consensus (posterior probability support under which nodes are collapsed in the final consensus tree). By default, the cutoff is equal to 0.5, although strictly speaking, standard significance thresholding would imply considering only those supports above a standard value of 0.95 as significant.
**readpb**

-`x <burn-in> [<every> <until>]`

defines the burn-in, the sub-sampling frequency, and the size of the samples of trees to be taken from the chain. The syntax is the same as for `bpcomp`.

-`c <cutoff>`

tunes the cutoff for the majority rule consensus. By default, the cutoff is equal to 0.5 (see `bpcomp`)

-`r` outputs the mean posterior rates at each site.

-`rr` outputs the mean posterior exchange rates.

-`ss` outputs the mean posterior profile at each site. with `-ps`: output the profiles as graphical logos.

-`m` outputs a histogram approximating the posterior distribution of the number of components.

-`cl` clustering of the profiles sampled during the MCMC: this is a single-link clustering, that has too many parameters:
  -`md <dist>` : the cutoff distance between profiles, that is, if the distance between two profiles is less than `<dist>`, they will end up in the same cluster, and
  -`ms <size>` : the minimum size of the clusters to be displayed in the final output, given that all clusters of size smaller than `<size>` will be merged together into a "garbage cluster". Defaults are `<dist> = 0.03` and `<size> = 10`.

-`s` save the subsample into a file. This can be useful if a chain is very big, and you want to subsample it to save disk space. Once a subsample has been saved, you can call `readpb` directly on it. For instance:

  `readpb -x 1000 100 11000 <chainname>`

  will save a sample of 100 points regularly spaced, taken from the chain `<chainname>`. This sample will be saved into a file called `<chainname>_sample.sample`. Then:

  `readpb -cl <chainname>_sample.sample`

  will perform a hierarchical clustering on this subsample.

-`ps`

activates the postscript output. If `readpb` is used for computing a consensus, a postscript tree will be produced (this requires LateX to be installed on your computer).
**readdiv**

`readdiv -x <burn-in> [every <until>] [other options] chainname`

This program works like **readpb**. It will output several files:
- `<chainname>_sample.dates`
- `<chainname>_sample.chronogram`
- `<chainname>_sample.labelss`

`-x <burn-in> [every <until>]`

defines the burn-in, the sub-sampling frequency, and the size of the samples of trees to be taken from the chain.

`-ps` activates the postscript output.
This program computes Bayes factor using thermodynamic integration. It works only under the normal approximation. The integration consists in running a quasi-static chain from one model to the other, and then back (see Lartillot and Philippe, 2006). The program stops automatically, and a confidence interval for the logarithm of the Bayes factor is proposed in the `<chainname>.log` file. The confidence interval accounts for the thermic lag and the discretization error, but not for the sampling error, which however, should be negligible, at least for the long integrations.

By default, the integration is between the specified relaxed clock model and the deconstrained model. The relaxed clock model can be specified using the same option as for `pb` (it is the log normal model by default). For instance:

```
bf -cov moc10.oes -ln moclnbf
```

computes the Bayes factor between the log-normal and the deconstrained model, for the `moc10.oes` dataset.

Two other types of integration can be performed:

- **-prior**

between the specified prior on divergence time (options to specify them are as in `pb`) and the uniform prior on divergence times.

- **-auto**

between the two auto-correlated models (CIR and log-normal). A positive value means a support in favor of CIR over the log-normal model.

To control the precision (and thus, the duration) of the integration:

- **-short**

burn-in of 100, integration over 1000 points (takes a few hours)

- **-long**

burning of 1,000, integration over 10,000 points (takes about one day)

- **-vlong**

burning of 1,000, integration over 10,000 points, but saving every 10 (takes more than one week)
tree2ps

tree2ps [options] <treefile>

This program converts a tree in newick format (<treefile>) into a postscript tree.

-0 <output>

will output the postscript tree into <output>.ps.

-r <outgroup>

reroot the tree.

+l (-l)

with (without) branchlengths.

+p (-p)

with (without) posterior probabilities at nodes.
For each point of the sample, `ppred` simulates a replicate using the parameter values defined by this point. All replicates are written as standard datafiles. You can then evaluate any statistic of interest on them, as well as on the true original dataset, so as to compute the deviation between the observed and the mean value of $s$ under the null distribution, the p-value, defined as the number of times the value simulated from the posterior predictive distribution (the 'predicted' value) is more extreme than the value simulated under the posterior distribution (the 'observed' value), the z-score, etc.

There are three pre-defined posterior predictive tests. They are called using the `-div`, `-comp` and `-sat` options, and measure the biochemical site-specificity, the compositional homogeneity, and the saturation, respectively. When using one of these options of `ppred`, the replicates are not written into files. Instead, the program directly computes the test statistics on them, and only outputs the z-score and p-values. In addition, a special posterior predictive testing framework is available for clock models (`-clockrate` option)

**Biochemical specificity (option -div)**

Under this option, the test statistic is the mean number of distinct amino acids observed at each column (the mean is taken over all the columns). This test statistic is meant as a way of measuring how well the site-specific biochemical patterns are accounted for by a model (see Lartillot et al, 2007). Often, standards models such as WAG, JTT or GTR are rejected by this statistic, except CAT, which should normally not be rejected, or very weakly.

**Compositional homogeneity (option -comp)**

This test statistic measures the compositional deviation of each taxon. The deviation is measured as the sum over the 20 amino-acids of the absolute differences between the taxon-specific and global empirical frequencies. The program also uses a global statistic, which is the maximum deviation over the taxa. This alleviates multiple testing, but on the other hand, only gives you a global answer (i.e. whether there are significant compositional deviations, but not who is significantly deviating).

Neither CAT, nor the standard WAG, JTT or GTR models, can account for the non-stationarity of the equilibrium frequencies. Therefore, and as you will probably see by yourself, in most cases, all these models are rejected by true data. For the moment, there is not yet any efficient model against this potential problem. But two workable solutions may be explored:
- removing the most deviating genes from your concatenation : this requires running the test on each gene separately, and sorting the genes, by decreasing z-score.
- alternatively, you may try to remove the most deviating taxa. Actually, there is an option to do this in `ppred`:

```
../ppred -comp -t 2 -x 100 10 rnapoll
```
will output a dataset where all taxa whose compositional z-score is greater than 2 have been removed. This dataset can then be used to make a new run.

**Saturation test (option -sat)**

In phylogenetics, saturation refers to the problem of multiple substitutions at a given site. Such multiple substitutions blur the phylogenetic signal. In addition, they create spurious convergences between unrelated taxa. Under challenging conditions (poor taxonomic sampling, or fast evolving species), these convergences can be mistaken for true phylogenetic signal, thereby creating systematic errors.

A good statistical test, to see whether a model is likely to produce artifacts, is therefore to measure how well the model anticipates sequence saturation. If the model does not anticipate saturation correctly, then, it will tend to interpret spurious convergences as true phylogenetic signal, and as a consequence, will more likely create artifacts (Lartillot et al., 2007).

The saturation index is defined as the number of homoplasies (convergences and reversions) per sites. However, even assuming the topology is known, convergences and reversions cannot be assessed just by looking at a dataset, but require the full substitution history to be reconstructed. To do this, one uses stochastic substitution mapping. Technically, the test thus proceeds as follows:

```bash
./ppred -sat -x 100 10 rnapol1
```

for each parameter value sampled from the posterior distribution, (i.e. each point of the MCMC run), two stochastic mappings are performed: one conditional on the true data at the leaves ('observed' mapping), and one unconstrained ('predicted'). The saturation index is then measured for both, and the procedure is repeated for each point of the run (here, 1 every 10). Finally, the observed and the predicted distributions of the saturation index are summarised by their means and variances. The (posterior predictive) p-value is also indicated. The ppred program also makes two files containing the histograms of the two distributions. These histograms can then be jointly plotted using **gnuplot**, or some other program.

For instance, in the case of brpo.ali, the test was conducted under WAG:

- **observed homoplasy**: 3.74290 +/- 0.0858103
- **posterior predictive**: 3.32902 +/- 0.025057

and CAT:

- **observed homoplasy**: 4.92162 +/- 0.219208
- **posterior predictive**: 4.86584 +/- 0.44926

In the present case, it is clear that (1) WAG infers a much lower saturation of the data than CAT, and that (2) it predicts significantly less saturation than it observes. Both facts indicate that WAG tends to underestimates saturation, which in turn may cause systematic errors, or at least distorsions of the posterior probabilities. On the other hand, CAT seems to correctly account for the observed saturation.
Testing branch lengths induced by clock relaxation (option -clockrate)

Under this option, for each parameter value sampled from the posterior distribution the program produces two trees: one with the branch lengths specified by that specific parameter value (thus, branch lengths taken from the posterior distribution), and saved in the .postlength file, and one with posterior predictive branch lengths: that is, conditional on the current times and parameters, a complete rate history is simulated along the tree, and the branch lengths induced by this rate history are computed, and written in the .predlength file. In this way, patterns of branch lengths induced by the model can be compared with the patterns directly inferred from the data.
Depending on the exact options, the `ancestral` program can either reconstruct ancestral sequences, or simulate complete substitution histories.

By default, `ancestral` will reconstruct the ancestral sequences. More precisely, it will estimate the posterior probability of each possible character state, at each node, and for each position of the alignment, and output those probabilities in a series of separate files, one for each node of the tree. The format is as follows:

```
<Nsite> <Nstate> [ <ALPHABET> ]
<site_index> <prob_1> <prob_2> ... <prob_Nstate>
...
```

Thus, on the first line, the total number of sites of the alignment, followed by the number of states (e.g. 20 for protein data), and the corresponding alphabet. Then, one line for each site, each displaying the index of each site, followed by the posterior probabilities of each possible character state, in the order specified by the first line.

The file itself is named as follows:
```
<chainename>_sample_<node_number>_<taxon1>_<taxon2>.ancstateprob
```

Concerning node numbers, a tree in which each node is labelled with its number is also provided as a separate file named `<chainename>_sample.labels`. For convenience, two taxa are also specified, of which the target node is the last common ancestor (one should be careful about how the tree is rooted here). Note that files are also produced for leaf nodes. This can be informative in the case where there are missing data: those data are then simply imputed using the posterior probabilities.

When used with the `-sub` option, `ancestral` will sample substitution histories from the posterior distribution. The substitution mappings are written into one single (large) file, using an extension of the newick format. Here is an example of a substitution mapping for a subtree of 3 taxa, Tax1, Tax2 and Tax3:

```
((Tax1_L:0.4:I:0.6:M:0.5:L,Tax2_L:0.7:L)L:0.6:L,Tax3_L:1.8:L)L
```

and a 'semi-graphical' explanation of its meaning (the branch lengths are such that one '-' corresponds to a length of 0.1):

```
|
|------------------ (L) Tax3
|
| (L)
|
|------- (L) Tax2
|
|------ (L)
|
|----/--/----/-- (I) Tax1
|   L->M M->I
```
Thus, the substitution mapping is such that the ancestor is in state L, and 2 substitutions have occurred along the branch leading to Tax1, successively, a transition from L to M, and a transition from M to I.

The file `<chainename>_sample.sub` produced by the program contains one such tree for each site (preceded by the index of the site), and this is repeated for each point of the sample:

```
point 1
site_1 substitution_tree
csite_2 substitution_tree
...
site_N substitution_tree

point 2
csite_1 substitution_tree
csite_2 substitution_tree
...
csite_N substitution_tree

...
```

You can get several independent substitution histories for each point, using the `-nrep <nrep>` option. You can also get substitution histories sampled from the posterior predictive distribution, in addition to those sampled from the posterior distribution, using the `-ppred` option. And finally, you can restrict the mapping to a subset of sites (an interval). Thus, for instance,

```
ancestral -x <burn-in> [<every> <until>] -sub -nrep 2 -i 3 5 name
```

will produce a file with the following structure:

```
point 1
rep 1
cposterior
csite_3 post_substitution_tree
csite_4 post_substitution_tree
csite_5 post_substitution_tree
cposterior predictive
csite_3 ppred_substitution_tree
csite_4 ppred_substitution_tree
csite_5 ppred_substitution_tree

rep2

cposterior
csite_3 post_substitution_tree
csite_4 post_substitution_tree
csite_5 post_substitution_tree
cposterior predictive
csite_3 ppred_substitution_tree
csite_4 ppred_substitution_tree
csite_5 ppred_substitution_tree

point 2
...
```
7. Cross-Validation

Cross-validation (CV) is a very general and reliable method for comparing models. The rationale is as follows: the dataset is randomly split into two (unequal) parts, the learning set and the test set. The parameters of the model are estimated on the learning set (i.e. the model is 'trained' on the learning set), and these parameter values are then used to compute the likelihood of the test set (which measures how well the test set is 'predicted' by the model). The overall procedure has to be repeated (and the resulting log likelihood scores averaged) over several random splits.

CV is computationally heavy. Note also that it does not work if the continuous gamma or the Dirichlet process on the site-specific rates is activated: you have to be under the discrete gamma model, or the uniform rate model. On the other hand, CV can be easily parallelized, as each replicated random split can be processed independently. To achieve an optimal parallel processing, a series of distinct programs are proposed in PhyloBayes, corresponding to each step of the method:
- prepare the replicates, using cvrep;
- run each model under each replicated learning set, using pb;
- compute the cross-validated log likelihood scores (i.e. the likelihood of each test set, averaged over the parameter values estimated by pb on the corresponding learning set), using readcv;
- pool all the cv-scores, and combine them into a global scoring of the models, using sumcv.

First, to prepare the replicated splits:

```
cvrep -nrep 10 -nfold 10 -d brpo.ali cvb
```

Here, -nrep 10 means that we are asking for 10 replicates, and -nfold 10 that we want to perform 10-fold cross validation: that is, for each replicate, the learning set will amount to 9/10, and the test set for 1/10, of the initial dataset. The replicates are saved under files using the specified prefix (here, cvb):

```
cvb0_learn.ali, cvb0_test.ali,
cvb1_learn.ali, cvb1_test.ali,
...
```

Next, you run a MCMC under each learning set, and for each model you want to compare, using pb. It is highly recommended to do these runs under a fixed topology (you can use the topology estimated by the model itself on the full dataset): this will be much faster. For instance, if you want to compare WAG and CAT:

```
pb -d cvb0_learn.ali -T brpo.tree -x 1 1100 CATcvb0_learn.ali
pb -d cvb0_learn.ali -T brpo.tree -x 1 1100 -wag WAGcvb0_learn.ali
```

and of course, the same should be done for the other 9 learning sets.

**IMPORTANT**: the name of the chains should be defined as here above, by prefixing the name of the dataset with some prefix standing for the model (e.g. CATcvb0_learn.ali). This is necessary for further processing of the resulting chains.

Once the chains have proceeded for all the replicates and all the models, you can compute, for each of
them, the cross-validated likelihoods, using \texttt{readcv}.

\begin{verbatim}
readcv -nrep 10 -x 100 1 CAT cvb
readcv -nrep 10 -x 100 1 WAG cvb
\end{verbatim}

The cross-validation score, for each replicate, is the likelihood under the test set, averaged over the posterior distribution of the learning set. Here, it is approximated by averaging over the parameter values visited by the chain run on the learning set, discarding a burnin of 100 points, and taking every point thereafter. The logarithm of the resulting average is stored into a file with a \texttt{.cv} extension. There is one such file for each replicate (thus, \texttt{cvb0.cv}, \texttt{cvb1.cv}, ..., \texttt{cvb9.cv}).

Note that \texttt{readcv} will process each replicate successively, which may take a very long time. If you want to get a first rough idea of the score, do not hesitate to subsample (e.g. \texttt{-x 100 10}). Also, if you are working on a multiprocessor, \texttt{readcv} allows you to parallelize the likelihood computation procedure, in several ways. First, you can explicitly call \texttt{readcv} on only one replicate:

\begin{verbatim}
readcv -rep 2 -x 100 10 CAT cvb
\end{verbatim}

This command will process only the replicate number 2. This allows you to split the work across several computers, according to what is most convenient for you. Second, you have the \texttt{-bg} (background) option, which will directly send the 10 jobs in parallel, all in the background mode.

\begin{verbatim}
readcv -bg -nrep 10 -x 100 10 CAT cvb
\end{verbatim}

And if you are working on a 'qsub' grid, the following command will send the 10 jobs in the queue:

\begin{verbatim}
readcv -qsub -nrep 10 -x 100 10 CAT cvb
\end{verbatim}

Finally, if you just want to prepare the 10 scripts, without launching them:

\begin{verbatim}
readcv -qprep -nrep 10 -x 100 10 CAT cvb
\end{verbatim}

Once all the cross-likelihood scores, for all the models and all the replicates, have been obtained, you need to gather them, and compute summary statistics:

\begin{verbatim}
sumcv -nrep 10 WAG CAT cvb
\end{verbatim}

You can use this command for more than 2 models (provided that the mean cv-likelihoods are available, and are all stored into the relevant files, with the \texttt{.cv} extension):

\begin{verbatim}
sumcv -nrep 10 WAG CAT GTR cvb
\end{verbatim}

This last program will average the cv-log-likelihood scores over replicates, using the leftmost model (here WAG) as the reference.
8. Priors

Branch lengths

-lexp
bl ~ iid Exponential of mean $\mu$

-lgam
bl ~ iid Gamma of mean $\mu$ and variance $\varepsilon \mu^2$ (reduces to the exponential distribution when $\varepsilon = 1$).

$\mu$ ~ Exponential of mean 0.1
$\varepsilon$ ~ Exponential of mean 1

Rates across sites

-cgam
$r$ ~ iid Gamma of mean 1 and variance $1/\alpha$

-dgam <ncat>
r ~ iid discretized Gamma of mean 1 and variance $1/\alpha$

-ratecat
$r$ ~ Dirichlet process of granularity $\zeta$ and Base distribution Gamma of mean 1 and variance $1/\alpha$

$\alpha$ ~ Exponential of mean 1
$\zeta$ ~ Exponential of mean 1

Profiles

-ncat 1
$\pi$ ~ Uniform

-cat
$\pi$ ~ Dirichlet process of granularity $\eta$ and base distribution $G_0$

-statflat
$G_0$ = Uniform

-statfree
$G_0$ = generalized Dirichlet, of center $\pi_0$ and concentration $\delta$

$\pi_0$ ~ Uniform
$\delta$ ~ Exponential of mean $S$ (where $S$ is the number of states, 4 for nucleotides, 20 for amino acids)
$\eta$ ~ Exponential of mean 10.
Relative exchange rates

\( rr \sim \text{iid Exponential of mean 1} \)

Relaxed clock parameters

Rates are relative (i.e. dimensionless). They are multiplied by a global scaling factor, \( \nu \) (expressed in number of substitutions per site and per unit of relative time). See Lepage et al for more details.

\[-\ln \]
\( \rho \sim \text{lognormal autocorrelated process of variance parameter } \sigma^2 \) (\( \nu \) in Thorne et al, 1998)

\[-\text{cir} \]
\( \rho \sim \text{CIR process of variance parameter } \sigma^2 \) and autocorrelation parameter \( \theta \)

\[-\text{ugam} \]
\( \rho \sim \text{iid gamma of variance parameter } \sigma^2 \)

\( \nu \sim \text{Exponential of mean 1} \)
\( \sigma^2 \sim \text{Exponential of mean 1} \)

under CIR:
\( \theta \sim \text{Exponential of mean 1 with the additional constraint that } \theta > \sigma^2/2 \) (and not \( \theta > \sigma/2 \) as written in Lepage et al 2007)

Divergence dates

Divergence dates are relative (dimensionless): root has divergence 1, and leaves divergence 0. These relative dates are then multiplied by a global scale factor \( Scale \) (expressed in Million of years) which is tune by the \(-\text{rp} \) (root prior) option.

\[-\text{unitime} \]
\( t \sim \text{uniform} \)

\[-\text{dir} \]
\( t \sim \text{Dirichlet prior of concentration parameter } \chi \) (when \( \chi = 1 \), prior is uniform).
\( \chi \sim \text{Exponential of mean 1} \)

\[-\text{bd} \]
\( t \sim \text{birth death process of parameters } \lambda \) (birth rate), \( \mu \) (death rate) and \( \omega \) (sampling fraction). To avoid identifiability, we set \( p_1 = \lambda - \mu \), and \( p_2 = \lambda \omega \)
\( p_1 \sim \text{Exponential of mean 1} \)
\( p_2 \sim \text{Exponential of mean 1} \)

\( Scale \sim \text{Uniform over } R^+ \) by default: this is a pseudo prior. Override if possible.

\[-\text{rp} <\text{mean}> <\text{stdev}> \]
\( Scale \sim \text{Gamma of mean and standard deviations such as specified.} \)
Covarion model

The covarion parameters can be expressed as
rate from OFF to ON: \( s_{01} = \xi \cdot (1-p_0) \)
rate from ON to OFF: \( s_{10} = \xi \cdot p_0 \)
where \( \xi \) is the rate of the ON-OFF process, and \( p_0 \) the stationary probability of the OFF state.

\( \xi \sim \text{Exponential of mean 1} \)
\( p_0 \sim \text{Uniform over } [0,1] \)

Mixture of branch lengths

The mixture of branch length is emulated by using branch length multipliers \( \lambda_{ij} \), for site i and branch j.
if we denote by \( \Lambda_i \) the vector of multipliers at site i (the \( \lambda_{ij} \), for \( j=1..2P-3 \), then
\( \Lambda \sim \text{Dirichlet process of base distribution } H_0, \text{ and granularity } \beta \)
\( H_0 = \text{product of independent gamma of mean 1 and variance } 1/\phi \)
\( \phi \sim \text{Exponential of mean 1} \)
\( \beta \sim \text{Exponential of mean 1} \)

Gene-specific branch length multipliers

If we denote by \( \Delta_g \) the vector of multipliers for gene g
\( \Delta \sim \text{product of independent gamma of mean 1 and variance } 1/\varphi \)
\( \varphi \sim \text{Exponential of mean 1} \)
References


Rota Stabelli, O., Yang, Z. and Telford, M.J. (2009). MtZoa: a general mitochondrial amino acid substitutions model for animal evolutionary studies. Molecular Phylogenetic and Evolution, 52,


